Homework Quiz 8, §2.4#15, §2.5#42, Solution

§2.4#15 Prove $\lim_{x\to 1}(2x+3)=5$.

Let $\epsilon>0$ and choose $\delta=\epsilon/2$. If $0<|x-1|<\delta$ then

$$|2x+3-5|=|2x-2|=2|x-1|<2\delta=2\left(\frac{\epsilon}{2}\right)=\epsilon.$$

Thus $\lim_{x\to 1}(2x+3)=5$.

§2.5#42 Find values of a, b that make f continuous everywhere.

$$f(x)=\begin{cases}\frac{x^2-4}{x-2} & \text{if } x<2\\ ax^2-bx+3 & \text{if } 2\le x<3\\ 2x-a+b & \text{if } x\ge 3\end{cases}$$

The function is clearly continuous at all points except $x=2$ and $x=3$. We need $\lim_{x\to 2}f(x)=f(2)$ and $\lim_{x\to 3}f(x)=f(3)$ to insure continuity. This means we require,

$$\lim_{x\to 2^-}\left(\frac{x^2-4}{x-2}\right)=\lim_{x\to 2^+}(ax^2-bx+3)=4a-2b+3$$

$$\lim_{x\to 2^-}\left(\frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}\right)=2+2=4 \quad \therefore \boxed{4=4a-2b+3} - \text{I}$$

And,

$$\lim_{x\to 3^-}(ax^2-bx+3)=\lim_{x\to 3^+}(2x-a+b)$$

$$9a-3b+3=6-a+b \quad \therefore \boxed{10a=4b+3} - \text{II}$$

Notice I yields $8=8a-4b+6$ then we add II to obtain

$$\begin{array}{rl} & 8=8a-4b+6\\ + & 10a=4b+3\\ \hline & 10a+8=8a+9 \end{array} \rightarrow 2a=1 \Rightarrow a=1/2$$

Eq^n I now says $4=2-2b+3$
$\Rightarrow -1=-2b$
$\Rightarrow 1/2=b$

Thus $a=1/2$, $b=1/2$ make f continuous everywhere