

1.) Find the limit below:

$$\lim_{x \rightarrow \infty} \left(\frac{x-42}{2x+3} \right) = \lim_{x \rightarrow \infty} \left(\frac{1 - \frac{42}{x}}{2 + \frac{3}{x}} \right) = \boxed{\frac{1}{2}}$$

2.) Find the limit below:

$$\lim_{x \rightarrow -\infty} \left(\frac{x^2 - 42}{x} \right) = \lim_{x \rightarrow -\infty} \left(x - \frac{42}{x} \right) = \boxed{-\infty}$$

3.) Calculate the following indeterminate limit:

$$\lim_{x \rightarrow a} \left(\frac{1}{x-a} \left[\frac{1}{x} - \frac{1}{a} \right] \right) = \lim_{x \rightarrow a} \left[\frac{\frac{1}{x} - \frac{1}{a}}{x-a} \right]$$

$$= \lim_{x \rightarrow a} \left[\frac{\frac{a-x}{xa}}{x-a} \right]$$

$$= \lim_{x \rightarrow a} \left[\frac{-1}{xa} \right]$$

$$= \frac{-1}{a^2}$$

note $a-x = -(x-a)$
 so it cancels to leave
 the -1 .

Remark: this calculation shows $f(x) = \frac{1}{x}$ has $f'(a) = \frac{-1}{a^2}$