

1.) Calculate $\frac{d}{dx} x^7$ and $\frac{d}{dx} \frac{1}{x^7}$.

$$\frac{d}{dx}(x^7) = \boxed{7x^6}$$

$$\frac{d}{dx}\left(\frac{1}{x^7}\right) = \frac{d}{dx}(x^{-7}) = \boxed{-7x^{-8}}$$

2.) Calculate $\frac{d}{dx} x(x^7 + \frac{1}{x})$.

$$\frac{d}{dx}(x^8 + 1) = \boxed{8x^7}$$

3.) Calculate $\frac{d}{dx} \frac{x + \sqrt[3]{x^2 - 4x}}{x^2}$.

$$\frac{d}{dx}\left(x^{-1} + x^{\frac{2}{3}-2} - 4x^{-1}\right) = -x^{-2} - \frac{4}{3}x^{-\frac{7}{3}} + 4x^{-2} = \boxed{3x^{-2} - \frac{4}{3}x^{-\frac{7}{3}}}$$

4.) If $f(x) = x^3 + 3x + 1$ then calculate $f'(2)$ and find the equation of the tangent line at $x = 2$.

$$f'(x) = 3x^2 + 3$$

$$f'(2) = 3(4) + 3 = 15 \Rightarrow y = f(2) + f'(2)(x-2)$$

$$\Rightarrow \boxed{y = 15 + 15(x-2)}$$

5.) Let $y = x^2 + \sqrt[3]{x} + 3^2$ and calculate $\frac{dy}{dx}$.

$$\boxed{\frac{dy}{dx} = 2x + \frac{1}{2\sqrt[3]{x}}}$$

6.) Let $f(x) = x^2 - 6x + 8$ calculate $f'(x)$ and make a sign-chart to find where $f(x)$ is increasing or decreasing. Graph the function and check your results.

$$f'(x) = 2x - 6 = 2(x-3)$$

$$f(x) = (x-2)(x-4)$$

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