LECTURE 20: GRADIENTS IN CURVELINEAR COORDINATES

$n=2$

$$\nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} = \langle f_x, f_y \rangle$$

$$\nabla f = \hat{r}\frac{\partial f}{\partial r} + \frac{1}{r}\hat{\theta}\frac{\partial f}{\partial \theta}$$

$$\hat{r} = \langle \cos\theta, \sin\theta \rangle$$
$$\hat{\theta} = \langle -\sin\theta, \cos\theta \rangle$$

$n=3$

$$\nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z} = \langle f_x, f_y, f_z \rangle$$

$$\nabla f = \hat{r}\frac{\partial f}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial f}{\partial \theta} + \hat{z}\frac{\partial f}{\partial z}$$

$$\nabla f = \hat{\rho}\frac{\partial f}{\partial \rho} + \frac{1}{\rho}\hat{\phi}\frac{\partial f}{\partial \phi} + \frac{1}{\rho\sin\phi}\hat{\theta}\frac{\partial f}{\partial \theta}$$

$$V = \frac{1}{\rho} \qquad f(\rho, \phi, \theta) = \frac{1}{\rho} \qquad \frac{\partial f}{\partial \phi} = \frac{\partial f}{\partial \theta} = 0$$

$$\frac{\partial f}{\partial \rho} = \frac{-1}{\rho^2}$$

$$\nabla V = \hat{\rho}\left(\frac{-1}{\rho^2}\right) + 0 + 0$$

$$\vec{E} = -\nabla V = \frac{\hat{\rho}}{\rho^2}$$