Lecture 32: Potential Formulation of Electromagnetism

Previously, in electrostatics $\nabla \times \vec{E} = 0$ thus $\vec{E} = -\nabla V$ was a reasonable construction. Now, with time varying electric and magnetic fields we face $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ hence $\nabla \times \vec{E} \neq 0$ generally. That said, since $\nabla \cdot \vec{B} = 0$ we are able to continue using $\vec{A}$ for the vector potential of $\vec{B}$; $\nabla \times \vec{A} = \vec{B}$. What's new here is we allow a t-dependence for $\vec{A}$.

Def/ Given $\vec{E}$ and $\vec{B}$ functions of $\vec{r}, t$ we say V is a scalar potential and $\vec{A}$ is a vector potential for the given fields provided

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

Remark: really $(V, \vec{A})$ is a package deal. They go together here, much like $\vec{E}$ & $\vec{B}$.

Thm/ Given $\vec{B} = \nabla \times \vec{A}$ the field $\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla V$ for some V.

Proof: we assume Maxwell's Eq's hold for $\vec{E}$ & $\vec{B}$,

$$\begin{aligned}
\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) &= \nabla \times \vec{E} + \nabla \times \frac{\partial \vec{A}}{\partial t} \\
&= -\frac{\partial \vec{B}}{\partial t} + \frac{\partial}{\partial t}(\nabla \times \vec{A}) \\
&= -\frac{\partial \vec{B}}{\partial t} + \frac{\partial \vec{B}}{\partial t} \\
&= 0 \implies \exists V \text{ for which}
\end{aligned}$$

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla V. //$$

(this justifies our formulation of $\vec{A}$ and V)

Th^m/ Given $V, \vec{A}$, if we define $\vec{B} = \nabla \times \vec{A}$ and $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$ then $\nabla \cdot \vec{B} = 0$ and $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

Furthermore, if $\nabla \cdot \vec{E} = \rho/\epsilon_0$ and $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ then V and $\vec{A}$ must solve,

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot \vec{A}) = \frac{-1}{\epsilon_0}\rho \qquad \text{(GRIFFITHS 10.4)}$$

$$\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} - \nabla\left(\nabla \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}\right) = -\mu_0 \vec{J} \qquad \text{(GRIFFITHS 10.5)}$$

Proof: $\nabla \cdot (\nabla \times \vec{A}) = 0 \therefore \nabla \cdot \vec{B} = 0$. Likewise,

$$\nabla \times \vec{E} = -\nabla \times \nabla V - \nabla \times \frac{\partial \vec{A}}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times \vec{A}) = -\frac{\partial \vec{B}}{\partial t}.$$

Then suppose we substitute $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$ into Gauss' Law $\nabla \cdot \vec{E} = \rho/\epsilon_0$,

$$\nabla \cdot \left(-\nabla V - \frac{\partial \vec{A}}{\partial t}\right) = \rho/\epsilon_0 \Rightarrow \boxed{\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot \vec{A}) = -\rho/\epsilon_0}.$$

Next, substitute $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$ and $\vec{B} = \nabla \times \vec{A}$ into Ampere's Law with Maxwell's correction $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\underbrace{\nabla \times (\nabla \times \vec{A})} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t}\left[-\nabla V - \frac{\partial \vec{A}}{\partial t}\right]$$

$$\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J} - \mu_0 \epsilon_0 \nabla\left(\frac{\partial V}{\partial t}\right) - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\therefore \boxed{\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2} - \nabla\left(\nabla \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}\right) = -\mu_0 \vec{J}}.$$

Gauge Freedom in Electromagnetism

Suppose $(V, \vec{A})$ and $(V', \vec{A}')$ produce the same $\vec{E}$ and $\vec{B}$ fields then

$$\underbrace{\nabla \times \vec{A} = \nabla \times \vec{A}'}_{} \quad \text{and} \quad -\nabla V - \frac{\partial \vec{A}}{\partial t} = -\nabla V' - \frac{\partial \vec{A}'}{\partial t}$$

$$\nabla \times (\vec{A}' - \vec{A}) = 0 \qquad \nabla(V' - V) = -\frac{\partial}{\partial t}(\vec{A}' - \vec{A})$$

$$\Rightarrow \underline{\vec{A}' - \vec{A} = \nabla \lambda}.$$

$$\Rightarrow \nabla(V' - V) = -\frac{\partial}{\partial t}(\nabla \lambda) = -\nabla\left(\frac{\partial \lambda}{\partial t}\right)$$

$$\Rightarrow \underline{V' - V = -\frac{\partial \lambda}{\partial t} + C}.$$

Absorbing the constant into $\frac{\partial \lambda}{\partial t}$ we find

$$\boxed{\begin{aligned} \vec{A}' &= \vec{A} + \nabla \lambda \\ V' &= V - \frac{\partial \lambda}{\partial t} \end{aligned}}$$

gauge transformation of potentials, these do not change the fields $\vec{E}$ & $\vec{B}$.

There are two common choices to freeze out the above freedom

Def^{n}/ Coulomb Gauge	$\nabla \cdot \vec{A} = 0$
Def^{n}/ Lorentz Gauge	$\nabla \cdot \vec{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$

COULOMB GAUGE $\nabla\cdot\vec{A}=0$

We've studied this already, $\nabla^2 V = \frac{-1}{\epsilon_0}\rho$ has solution

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}',t)}{\mathscr{r}}\,d\tau'$$

notice even when $\vec{r}$ and $\vec{r}'$ are vastly apart, the effect of the charge density ρ changing in time is <u>immediately</u> seen in $V(\vec{r},t)$. How can this be? (relativity forbids faster than light speed information travel)

$$\underline{\nabla^2\vec{A} - \mu_0\epsilon_0\frac{\partial^2\vec{A}}{\partial t^2} - \nabla\left(\cancelto{0}{\nabla\cdot\vec{A}} + \mu_0\epsilon_0\frac{\partial V}{\partial t}\right) = -\mu_0\vec{J}} \; *$$

$\Longrightarrow$ $\vec{A}$ will have t-dependence as well

$$\vec{E} = -\nabla V - \frac{\partial\vec{A}}{\partial t} \longleftarrow$$ has to build-in the delayed propagation of charge density in time

* no fun to solve.

LORENZ GAUGE (apparently no t here, different dude)

Notice $\nabla\cdot\vec{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$ simplifies * to

$$\boxed{\nabla^2\vec{A} - \mu_0\epsilon_0\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0\vec{J}}$$

On the other hand, $\nabla^2 V + \frac{\partial}{\partial t}(\nabla\cdot\vec{A}) = -\rho/\epsilon_0$ yields

that $\nabla^2 V + \frac{\partial}{\partial t}\left(-\mu_0\epsilon_0\frac{\partial V}{\partial t}\right) = -\rho/\epsilon_0$ hence,

$$\boxed{\nabla^2 V - \mu_0\epsilon_0\frac{\partial^2 V}{\partial t^2} = \frac{-1}{\epsilon_0}\rho}$$

$$\boxed{\begin{aligned}\Box^2\vec{A} &= -\mu_0\vec{J}\\ \Box^2 V &= -\rho/\epsilon_0\end{aligned}}$$

In short, $\nabla^2 - \mu_0\epsilon_0\frac{\partial^2}{\partial t^2} = \Box^2 \longleftarrow$ <u>d'Alembertian</u>

RETARDED POTENTIALS

We know solutions of the static $\nabla^2 V = \frac{-1}{\epsilon_0}\rho$ and $\nabla^2 \vec{A} = -\mu_0 \vec{J}$ are given by $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')\,d\tau'}{\mathscr{r}}$ and $\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}')\,d\tau'}{\mathscr{r}}$ where $\mathscr{r} = \|\vec{r} - \vec{r}'\|$ and $\vec{r}$ is the field point and $\vec{r}'$ is the source point. We now introduce the retarded time for message to travel in straight line from $\vec{r}'$ to $\vec{r}$ at speed c,

$$\boxed{t_r = t - \frac{\mathscr{r}}{c}}$$

In view of the above notation, the RETARDED POTENTIALS are given in terms of integrals of the source charge density and current density at retarded time

$$\boxed{\begin{aligned} V(\vec{r},t) &= \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}', t_r)}{\mathscr{r}}\,d\tau' \\ \vec{A}(\vec{r},t) &= \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}', t_r)}{\mathscr{r}}\,d\tau' \end{aligned}}$$

We claim the $(V, \vec{A})$ given above solve

(1.) $\nabla\cdot\vec{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$ (Lorenz Gauge Condition)

(2.) $\underbrace{\Box^2\vec{A} = -\mu_0\vec{J} \quad \text{and} \quad \Box^2 V = -\rho/\epsilon_0}_{\text{inhomogeneous wave eq}^{\text{n}}\text{s for } \vec{A}\ \&\ V.}$

Proof: Suppose $V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}',t_r)}{\mathscr{r}}\,d\tau'$

and $\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}}\,d\tau'$ where $t_r = t - \frac{\mathscr{r}}{c}$

and $\mathscr{r} = \| \vec{r} - \vec{r}' \|$

$$\nabla V = \frac{1}{4\pi\epsilon_0}\int \nabla\left(\frac{1}{\mathscr{r}}\rho\right) d\tau'$$

$$= \frac{1}{4\pi\epsilon_0}\int \left(\underline{\nabla\left(\frac{1}{\mathscr{r}}\right)\rho + \frac{1}{\mathscr{r}}\nabla\rho}\right) d\tau'$$

$$\nabla\rho = \nabla\left(\rho(\vec{r}',t_r)\right)$$

$$= \frac{\partial\rho}{\partial t_r}\nabla t_r$$

note $\vec{r}'$ independent of $\vec{r}$ which forms ∇
$\nabla = \hat{x}\,\partial_x + \hat{y}\,\partial_y + \hat{z}\,\partial_z$

$$= \dot{\rho}\left(-\frac{1}{c}\nabla\mathscr{r}\right)$$

$$= -\frac{1}{c}\dot{\rho}\,\hat{\mathscr{r}}$$

We also know that $\nabla\left(\frac{1}{\mathscr{r}}\right) = \frac{-1}{\mathscr{r}^2}\hat{\mathscr{r}}$ thus,

$$\nabla V = \frac{1}{4\pi\epsilon_0}\int \left[\frac{-\rho}{\mathscr{r}^2} - \frac{\dot{\rho}}{\mathscr{r}c}\right]\hat{\mathscr{r}}\,d\tau'$$

continued

$$\nabla\dot{\rho} = \nabla\left(\dot{\rho}(\vec{r}',t_r)\right)$$

$$= \frac{\partial\dot{\rho}}{\partial t_r}\nabla t_r$$

$$= -\frac{1}{c}\ddot{\rho}\,\hat{\mathscr{r}}$$ (save for later)

Beginning with $\nabla V = \frac{1}{4\pi\epsilon_0}\int \left[\frac{-\rho}{r^2} - \frac{\dot{\rho}}{r c}\right]\hat{r}\, d\tau'$

$$\nabla^2 V = \nabla \cdot \nabla V$$

$$= \frac{1}{4\pi\epsilon_0}\int \left[-\nabla\cdot\left(\frac{\rho}{r^2}+\frac{\dot{\rho}}{r c}\right)\hat{r}\right] d\tau'$$

$$= \frac{-1}{4\pi\epsilon_0}\int \left[\nabla\cdot\left(\rho\frac{\hat{r}}{r^2}\right) + \nabla\left(\frac{\dot{\rho}}{c}\frac{\hat{r}}{r}\right)\right] d\tau'$$

$$= \frac{-1}{4\pi\epsilon_0}\int \left[\underline{(\nabla\rho)}\cdot\frac{\hat{r}}{r^2} + \rho\,\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) + \frac{1}{c}\underline{\underline{(\nabla\dot{\rho})}}\cdot\frac{\hat{r}}{r} + \frac{\dot{\rho}}{c}\nabla\cdot\left(\frac{\hat{r}}{r}\right)\right] d\tau'$$

$$= \frac{-1}{4\pi\epsilon_0}\int \left[\underline{\frac{-1}{c}\dot{\rho}\,\hat{r}}\cdot\frac{\hat{r}}{r^2} + \rho\,\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) - \frac{1}{c^2}(\ddot{\rho}\,\hat{r})\cdot\frac{\hat{r}}{r} + \frac{\dot{\rho}}{c r^2}\right] d\tau'$$

$$= \frac{1}{4\pi\epsilon_0}\int \left[\frac{1}{c^2}\left(\frac{\ddot{\rho}}{r}\right) - 4\pi\rho\,\delta^3(\vec{r})\right] d\tau'$$

$$= \frac{1}{c^2}\left[\frac{1}{4\pi\epsilon_0}\int \frac{\ddot{\rho}}{r}\, d\tau'\right] - \frac{1}{\epsilon_0}\int \rho\,\delta^3(\vec{r})\, d\tau'$$

$$= \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left[\frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,', t_r)}{r}\, d\tau'\right] - \frac{\rho(\vec{r}\,', t)}{\epsilon_0}$$

$$= \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} - \frac{1}{\epsilon_0}\rho(\vec{r}, t)$$

$$\left(\begin{array}{c} t_r = t - \frac{1}{c} r \\ \frac{\partial}{\partial t_r} = \frac{\partial}{\partial t} \end{array}\right)$$

Remark: it remains to show

$\Box^2 \vec{A} = -\mu_0 \vec{J}$ and $\nabla\cdot\vec{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$

$\boxed{E1}$ An infinite straight wire carries current

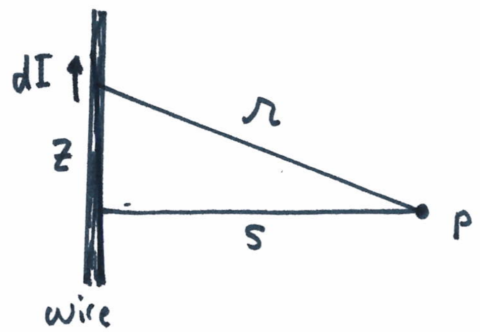
$$I(t) = \begin{cases} 0 & \text{for } t \le 0, \\ I_0 & \text{for } t > 0. \end{cases}$$

Find resulting electric and magnetic fields

Here $V = 0$ since we assume the wire is neutral

Let's place the current flow on the z-axis,

$$\vec{A}(s,t) = \frac{\mu_0}{4\pi}\hat{z}\int_{-\infty}^{\infty}\frac{I(t_r)}{\mathscr{r}}\,dz$$



$t < s/c$ no "news" to P $\rightarrow$ $\underline{\vec{A}(s,t) = 0}$.

$t > s/c$ signal reached P

<u>For $t > s/c$</u>,

$$\vec{A}(s,t) = \left(\frac{\mu_0 I_0}{4\pi}\hat{z}\right)\int_{-\sqrt{(ct)^2 - s^2}}^{\sqrt{(ct)^2 - s^2}}\frac{dz}{\sqrt{s^2+z^2}}$$

$$= \frac{\mu_0 I_0}{2\pi}\hat{z}\int_0^{\sqrt{(ct)^2-s^2}}\frac{dz}{\sqrt{s^2+z^2}}$$

$$= \frac{\mu_0 I_0 \hat{z}}{2\pi}\left[\ln\left(\sqrt{s^2+z^2} + z\right)\right]\Bigg|_0^{\sqrt{(ct)^2-s^2}}$$

$$= \frac{\mu_0 I_0}{2\pi}\ln\left(\frac{ct + \sqrt{(ct)^2 - s^2}}{s}\right)\hat{z}$$

E1 continued

We calculated the retarded potentials, $V = 0$,

$$\vec{A}(s,t) = \frac{\mu_0 I_0}{2\pi} \ln\left(\frac{ct + \sqrt{(ct)^2 - s^2}}{s}\right)\hat{z}$$

Then we calculate fields from $\vec{B} = \nabla \times \vec{A}$ and $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} = -\frac{\partial \vec{A}}{\partial t}$. Use curl in cylindrical coordinates formula, since $A_\phi = 0$ and $A_s = 0$,

$$\nabla \times \vec{A} = \frac{1}{s}\frac{\partial A_z}{\partial \phi}\hat{s} - \frac{\partial A_z}{\partial s}\hat{\phi}$$

$$= -\frac{\mu_0 I_0}{2\pi}\frac{\partial}{\partial s}\left[\ln\left(ct + \sqrt{(ct)^2 - s^2}\right) - \ln(s)\right]\hat{\phi}$$

$$= \frac{\mu_0 I_0}{2\pi}\left[\frac{1}{s} - \left(\frac{1}{ct + \sqrt{(ct)^2 - s^2}}\right)\left(\frac{-2s}{2\sqrt{(ct)^2 - s^2}}\right)\right]\hat{\phi}$$

$$= \frac{\mu_0 I_0}{2\pi s}\left[1 + \frac{s^2}{ct\sqrt{(ct)^2 - s^2} + (ct)^2 - s^2}\right]\hat{\phi}$$

$$= \frac{\mu_0 I_0}{2\pi s}\left[\frac{ct\sqrt{(ct)^2 - s^2} + (ct)^2 - s^2 + s^2}{ct\sqrt{(ct)^2 - s^2} + (ct)^2 - s^2}\right]$$

$$= \frac{\mu_0 I_0}{2\pi s}\left[\frac{ct\sqrt{(ct)^2 - s^2} + (ct)^2}{ct\sqrt{(ct)^2 - s^2} + (ct)^2 - s^2}\right]$$

$$= \frac{\mu_0 I_0}{2\pi s}\left[\frac{ct}{\sqrt{(ct)^2 - s^2}}\right]\hat{\phi}$$

(see over for another attempt) ↷

E1 continued

I thought maybe I had a mistake, but, no it's just algebra. The last couple lines were tricky,

$$\nabla \times \vec{A} = -\frac{\partial A_s}{\partial s}\hat{\phi}$$

$$= -\frac{\mu_0 I_0}{2\pi}\frac{\partial}{\partial s}\left[\ln(u)\right]\hat{\phi} \qquad u = \frac{ct + \sqrt{(ct)^2 - s^2}}{s}$$

$$= \left(-\frac{\mu_0 I_0}{2\pi}\hat{\phi}\right)\left[\frac{1}{u}\frac{\partial}{\partial s}\left[\frac{ct + \sqrt{(ct)^2 - s^2}}{s}\right]\right]$$

$$= \left(-\frac{\mu_0 I_0}{2\pi}\hat{\phi}\right)\left[\frac{1}{u}\left[\frac{\frac{-s}{\sqrt{(ct)^2 - s^2}}\cdot s - \left(ct + \sqrt{(ct)^2 - s^2}\right)}{s^2}\right]\right]$$

$$= \left(-\frac{\mu_0 I_0}{2\pi}\hat{\phi}\right)\left(\frac{s}{ct + \sqrt{(ct)^2 - s^2}}\right)\left(\frac{1}{s^2}\right)\left(\frac{-s^2}{\sqrt{(ct)^2 - s^2}} - \left(ct + \sqrt{(ct)^2 - s^2}\right)\right)$$

$$= \left(\frac{\mu_0 I_0}{2\pi s}\hat{\phi}\right)\left(ct + \sqrt{(ct)^2 - s^2} + \frac{s^2}{\sqrt{(ct)^2 - s^2}}\right)\left(\frac{1}{ct + \sqrt{(ct)^2 - s^2}}\right)$$

$$= \left(\frac{\mu_0 I_0}{2\pi s}\hat{\phi}\right)\left[1 + \frac{s^2}{(ct)^2 - s^2 + ct\sqrt{(ct)^2 - s^2}}\right]$$

$$= \left(\frac{\mu_0 I_0}{2\pi s}\hat{\phi}\right)\left(\frac{(ct)^2 - s^2 + ct\sqrt{(ct)^2 - s^2} + s^2}{(ct)^2 - s^2 + ct\sqrt{(ct)^2 - s^2}}\right)$$

$$= \left(\frac{\mu_0 I_0}{2\pi s}\hat{\phi}\right)\left(\frac{ct\left(ct + \sqrt{(ct)^2 - s^2}\right)}{\sqrt{(ct)^2 - s^2}\left(\sqrt{(ct)^2 - s^2} + ct\right)}\right)$$

$$= \frac{\mu_0 I_0}{2\pi s}\frac{ct}{\sqrt{(ct)^2 - s^2}}\hat{\phi}$$

E1 continued

After some calculation we found $\vec{B} = \frac{\mu_0 I_0}{2\pi s}\frac{ct}{\sqrt{(ct)^2 - s^2}}\hat{\phi}$

Next we find $\vec{E}$,

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} \qquad (\text{since } V = 0 \text{ here})$$

$$= -\frac{\partial}{\partial t}\left[\frac{\mu_0 I_0}{2\pi}\left[\ln\left(ct + \sqrt{(ct)^2 - s^2}\right) - \underbrace{\ln(s)}_{0}\right]\hat{z}\right]$$

$$= \frac{-\mu_0 I_0}{2\pi}\left(\frac{1}{ct + \sqrt{(ct)^2 - s^2}}\right)\left(c + \frac{2c^2 t}{2\sqrt{(ct)^2 - s^2}}\right)\hat{z}$$

$$= \frac{-\mu_0 I_0 c}{2\pi}\left[\frac{1}{ct + \sqrt{(ct)^2 - s^2}}\right]\left[\frac{\sqrt{(ct)^2 - s^2} + ct}{\sqrt{(ct)^2 - s^2}}\right]\hat{z}$$

$$= \frac{-\mu_0 I_0 c}{2\pi\sqrt{(ct)^2 - s^2}}\hat{z}$$

In summary, there is a cylinder of nontrivial fields of radius $s = ct$ in which the fields expand.

$$\left.\begin{aligned} \vec{E} &= \frac{-\mu_0 I_0 c}{2\pi\sqrt{(ct)^2 - s^2}}\hat{z} \\ \vec{B} &= \left(\frac{\mu_0 I}{2\pi s}\right)\left[\frac{ct}{\sqrt{(ct)^2 - s^2}}\right]\hat{\phi} \end{aligned}\right\} \text{ for } s < ct$$

$$\vec{E}, \vec{B} = 0 \; \} \text{ for } s > ct \text{ where } \vec{A}(s,t), V(s,t) \text{ both zero.}$$

<u>Remark</u>: as $ct \longrightarrow \infty$ notice $\vec{E} \longrightarrow 0$ whereas $\vec{B} \longrightarrow \frac{\mu_0 I}{2\pi s}\hat{\phi}$.