LECTURE 35 : CONSTANT VELOCITY FIELDS & KINK RADIATION MODEL

Let us recall the formulas for $\vec{E}$ and $\vec{B}$ due to a point charge q with position $\vec{w}(t)$,

$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left[(c^2-v^2)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right]$$

$$\vec{B}(\vec{r},t) = \frac{1}{c}\hat{\mathscr{r}}\times\vec{E}(\vec{r},t)$$

where $\vec{u} = c\hat{\mathscr{r}} - \vec{v}$ and $\vec{v}, \vec{a}, \vec{\mathscr{r}}$ are evaluated at the retarded time t_r given by $\underline{\|\vec{r}-\vec{w}(t_r)\| = c(t-t_r)}$

Let us consider a point charge q with constant velocity

$$\vec{a} = 0 \quad\Rightarrow\quad \vec{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{(c^2-v^2)\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\right]\vec{u}$$

Supposing $\vec{w}(0)=0$ and $\vec{a}=0$ we find $\vec{w} = t\vec{v}$

$$\mathscr{r}\vec{u} = c\vec{\mathscr{r}} - \mathscr{r}\vec{v} \qquad (\vec{\mathscr{r}} = \vec{r}-\vec{w}(t_r)) \qquad \underline{\mathscr{r} = c(t-t_r)}.$$

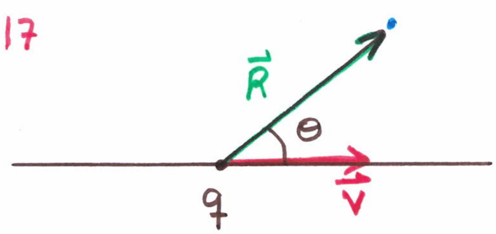
$$= c(\vec{r}-t_r\vec{v}) - c(t-t_r)\vec{v}$$

$$= c(\vec{r}-t\vec{v})$$

Recall, for constant velocity we worked out

$$\mathscr{r}c - \vec{\mathscr{r}}\cdot\vec{v} = \vec{\mathscr{r}}\cdot\vec{u} = \sqrt{(c^2t-\vec{r}\cdot\vec{v})^2 + (c^2-v^2)(r^2-c^2t^2)}$$

$$= \underbrace{Rc\sqrt{1-\frac{v^2}{c^2}\sin^2\theta}}_{\text{Problem 10.17}} \qquad (\vec{R} = \vec{r}-t\vec{v})$$



$$\vec{R}\cdot\vec{v} = vR\cos\theta = (\vec{r}-t\vec{v})\cdot\vec{v}$$

$$\underline{vR\cos\theta = \vec{r}\cdot\vec{v} - tv^2}.$$

Consider the expression under $\sqrt{\quad}$, $\underline{t\vec{v} = \vec{r} - \vec{R}}$

$$\mathcal{B} = (c^2t - \vec{r}\cdot\vec{v})^2 + (c^2 - v^2)(r^2 - c^2t^2) =$$

$$= \cancel{c^4t^2} - 2c^2t(\vec{r}\cdot\vec{v}) + (\vec{r}\cdot\vec{v})^2 + c^2r^2 - \cancel{c^4t^2} - v^2r^2 + v^2c^2t^2$$

$$= (\vec{r}\cdot\vec{v})^2 + (c^2 - v^2)r^2 + c^2(vt)^2 - 2c^2\,\vec{r}\cdot t\vec{v}$$

$$= (\vec{r}\cdot\vec{v})^2 + (c^2 - v^2)r^2 + c^2(\vec{r} - \vec{R})\cdot(\vec{r} - \vec{R}) - 2c^2\,\vec{r}\cdot(\vec{r} - \vec{R})$$

cancel

$$= (\vec{r}\cdot\vec{v})^2 + (c^2 - v^2)r^2 + c^2(r^2 + R^2 - 2\vec{r}\cdot\vec{R}) - 2c^2(r^2 - \vec{r}\cdot\vec{R})$$

cancel

survivors.

$$= (\vec{r}\cdot\vec{v})^2 - r^2v^2 + c^2R^2$$

However, we can calculate

$$(\vec{r}\cdot\vec{v})^2 - r^2v^2 = [(\vec{R} + t\vec{v})\cdot\vec{v}]^2 - (\vec{R} + t\vec{v})\cdot(\vec{R} + t\vec{v})\,v^2$$

$$= (\vec{R}\cdot\vec{v} + tv^2)^2 - (R^2 + 2t\vec{R}\cdot\vec{v} + t^2v^2)v^2$$

$$= (\vec{R}\cdot\vec{v})^2 + \cancel{2(\vec{R}\cdot\vec{v})tv^2} + \cancel{t^2v^4} - R^2v^2 - \cancel{2t(\vec{R}\cdot\vec{v})v^2} - \cancel{t^2v^4}$$

$$= R^2v^2\cos^2\theta - R^2v^2$$

$$= -R^2v^2\sin^2\theta$$

$$\therefore\quad \mathcal{B} = -R^2v^2\sin^2\theta + c^2R^2 = R^2c^2\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)$$

$$\therefore\quad \underline{\vec{\mathscr{r}}\cdot\vec{u} = Rc\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}}$$

(Problem 10.17 of 5th Edition)

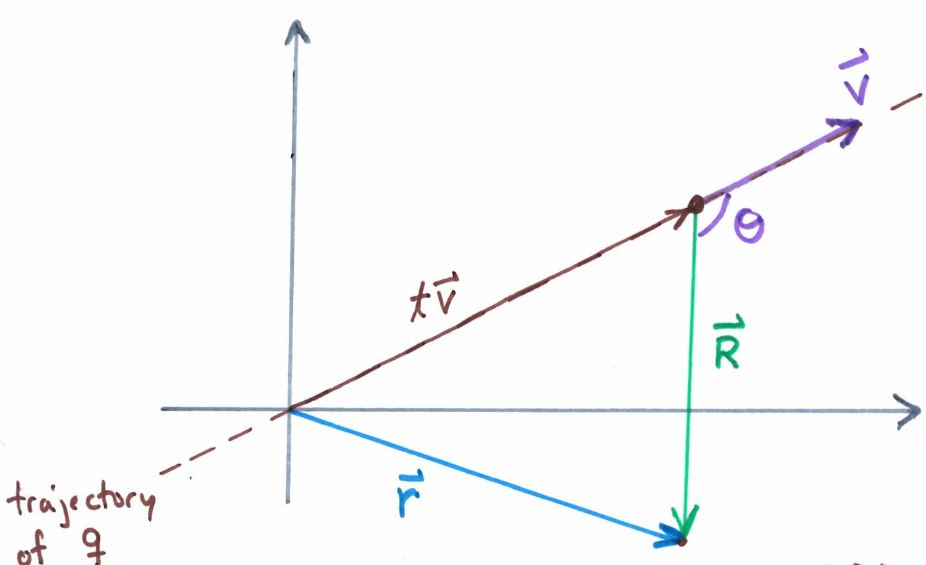
We find that, notice $\mathfrak{r}\vec{u} = c(\vec{r} - vt)$ and $\vec{R} = \vec{r} - vt$ hence,

$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0}\frac{(c^2 - v^2)\mathfrak{r}}{\left(Rc\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta}\right)^3}\vec{u}$$

$$= \frac{q}{4\pi\epsilon_0}\frac{(c^2 - v^2)c(\vec{r} - \vec{v}t)}{R^3c^3\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}$$

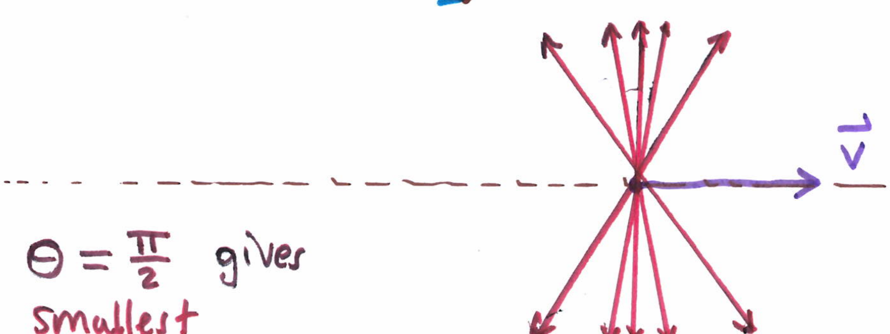
$$\boxed{\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0}\frac{1 - v^2/c^2}{\left(1 - \frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}\frac{\hat{R}}{R^2}}$$

Remark: $\vec{R} = \vec{r} - \vec{v}t$ is vector from the present location of q to the field point $\vec{r}$.



$$t\vec{v} + \vec{R} = \vec{r}$$

$$\underbrace{\vec{R} = \vec{r} - t\vec{v}}_{\text{points from present location of } q \text{ to the field point } \vec{r}}$$



$\theta = \frac{\pi}{2}$ gives smallest denominator hence largest $E = \frac{q}{4\pi\epsilon_0}\gamma\frac{\hat{R}}{R^2}$ where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Remark: $\theta = 0, \pi$ gives $\underbrace{\vec{E} = \frac{q}{4\pi\epsilon_0}(1 - v^2/c^2)\frac{\hat{R}}{R^2}}$

- enhanced in directions $\perp$ to $\vec{v}$ by factor $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$
- reduced in $\pm\vec{v}$ direction

Magnetic Field

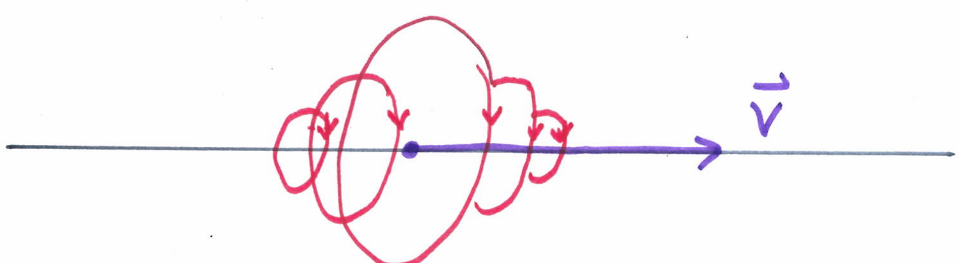
$$\hat{\mathscr{r}} = \frac{\vec{r} - t_r\vec{v}}{\mathscr{r}} = \frac{(\vec{r} - t\vec{v}) + (t - t_r)\vec{v}}{\mathscr{r}} = \frac{\vec{R}}{\mathscr{r}} + \frac{\vec{v}}{c}$$

$$\vec{B} = \frac{1}{c}(\hat{\mathscr{r}} \times \vec{E}) = \frac{1}{c^2}(\vec{v} \times \vec{E})$$

When $v^2 << c^2$ we find that

$$\vec{E}(\vec{r}, t) \approx \frac{q}{4\pi\epsilon_0}\frac{\hat{R}}{R^2}$$

$$\vec{B}(\vec{r}, t) \approx \frac{\mu_0}{4\pi}\frac{q}{R^2}(\vec{v} \times \hat{R})$$



$\vec{B}$ circle around trajectory

THOMSON KINK MODEL

A point charge is at rest at the origin when it is given a kick in the x-direction. Then the charge moves at constant velocity $\vec{V} = v\hat{x}$.

Assume kick is very short in duration then we can think of charge $\vec{V} = \begin{cases} 0 & t < 0 \\ v\hat{x} & t > 0 \end{cases}$.

Outside sphere $r_2 = ct$ the "news" of kick not arrived

Inside sphere $r_1 = c(t-\tau)$ we have the constant velocity electric field.

radiation zone $r_1 < r < r_2$ shell of thickness $c\tau$.