LECTURE 36 : RADIATION (TO INFINITY)

When charges accelerate they radiate (usually). This means energy is lost from a localized accelerated source. Imagine a giant sphere,

$$P(r,t) = \oint \vec{S}\cdot d\vec{a} = \frac{1}{\mu_0}\oint (\vec{E}\times\vec{B})\cdot d\vec{a}$$

This energy travels at light speed, hence the power recieved at time t is due to source behavior at the retarded time $t_0 = t - r/c$. Thus,

$$P_{radiated}(t_0) = \underbrace{\lim_{r\to\infty} P(r, t_0 + r/c)}_{\text{energy per unit time which is carried away never to return}}$$

<u>Approximation</u>, or asymptotic behavior is key. For static electric & magnetic fields,

$$E \sim \frac{1}{r^2} \qquad B \sim \frac{1}{r^2}$$

$$\|\vec{E}\times\vec{B}\| \sim \frac{1}{r^4} \quad \Rightarrow \quad \underline{P_{radiated} = 0}.$$

<u>Sketch</u>: $d\vec{a} = (r^2 da)\hat{r}$ so the integral goes like $\frac{1}{r^2}$ integrated to ∞ which is zero.

ELECTRIC DIPOLE RADIATION:

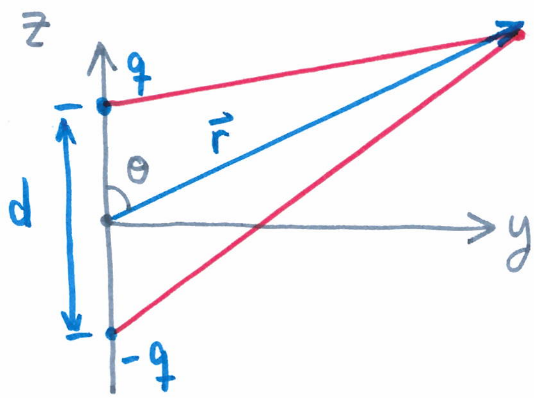
Imagine charge q_0 flowing back & forth between two metal spheres, distance d apart, at frequency ω

$$q(t) = q_0 \cos(\omega t)$$

$$\vec{p}(t) = p_0 \cos(\omega t)\,\hat{z} \quad \text{where } p_0 = q_0 d$$

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\left\{ \frac{q_0 \cos(\omega(t - \mathscr{r}_+/c))}{\mathscr{r}_+} - \frac{q_0 \cos(\omega(t - \mathscr{r}_-/c))}{\mathscr{r}_-} \right\}$$

$$\mathscr{r}_\pm = \sqrt{r^2 \mp rd\cos\theta + (d/2)^2}$$



approximations

1: $d << r$

2: $d << c/\omega$

3.) $r >> c/\omega$

After some calculation (p. 474–475)

$$V(r,\theta,t) = \frac{-p_0\omega}{4\pi\epsilon_0 r}\left(\frac{\cos\theta}{r}\right)\sin[\omega(t - r/c)]$$

$$\vec{A}(r,\theta,t) = \frac{-\mu_0 p_0 \omega}{4\pi r}\sin[\omega(t - r/c)]\,\hat{z}$$

Next, $\vec{E} = -\nabla V - \dfrac{\partial \vec{A}}{\partial t}$

$\vec{B} = \nabla \times \vec{A}$