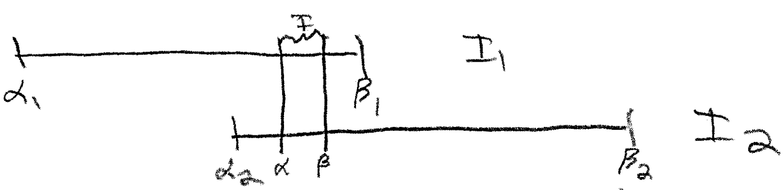
2/ 10/25

(3) THE DEF ON (2) DOES NOT DEPEND ON CHOICE OF WHICH I WE CHOOSE TO USE. (USE LEMMA 1)
(i.e. IF $t \in I_1 \cap I_2$ THEN $\varphi_1(t) = \varphi_2(t)$ FOR $t \in I_1 \cap I_2$)

(4) $x(t)$ SATISFIES IVP $\forall t \in J$
Pf: IF $t \in J$ THEN $t_1 \in I_1$, $\varphi_1(t)$ DEFINED ON I_1
$\varphi(t)$ IS SOL. ON I_1
$\Rightarrow x(t)$ IS SOL. ON I_1

(5) WE DO NOT USE CLOSED INTERVALS
CAN EXTEND SOLUTION TO $[\beta - a, \beta + a]$

p89. E OPEN $\subset \mathbb{R}^n$ $x_0 \in E$ $f \in C^1(E)$
$u_1(t)$ & $u_2(t)$ ARE SOL. ON I_1 & I_2 RESP.
THEN $u_1(t) = u_2(t)$ $\forall t \in I_1 \cap I_2$



SHOW $u_1 = u_2$ ON (α_2, β_1)
PROOF: (i) THE OPEN INTERVALS I S.t. $u_1 = u_2$ IS NONEMPTY $(-a, a)$!
(ii) IF (α, β) IS MAXIMAL INTERVAL
ASSUME $\beta < \beta_1$, THEN $u_1 = u_2$ AT β
IVP w/ $x(\beta) = u_1(\beta) \Rightarrow$ $a < \beta_1 < \beta$
HAS ! SOL ON $(\beta - a, \beta + a)$
SO $I = I_1 \cap I_2$ & $u_1 = u_2$ ON I $\Rightarrow$

1/ MAXIMAL INTERVAL OF EXISTENCE 10/27

EITHER $\beta = \infty$ OR $x(t)$ LEAVES ANY COMPACT $K \subset E$

Pf: ASSUME $\beta < \infty$ & $x(t)$ STAYS IN K $\Rightarrow$ $\Downarrow$

ASSUME $[0, \beta)$

SHOW $\lim_{t \to \beta^-} x(t)$ EXISTS DEFINE IT TO BE $x(\beta)$

TAKE $x(t_n)$ $t_n \to \beta$

DEFINE $x(t_n)$ USING INTEGRAL EQ.

$x(t_1) = x_0 \int_0^{t_1} f(x(s))ds$

$x(t_2) = x_0 \int_0^{t_2} f(x(s))ds$

$|x(t_1) - x(t_2)| \leq \int_{t_1}^{t_2} |f(x(s))|ds \leq M|t_2 - t_1|$

$M = \sup_{x \in K} f(x)$

$|x(t_n) - x(t_m)|$ CONV. SINCE t_n CONV.

$\Rightarrow x(t_n)$ HAS LIMIT

$x_1 = \lim_{t \to \beta^-} x(t)$

$u(t) = \begin{cases} x(t) & 0 \leq t < \beta \\ x_1 & t = \beta \end{cases}$ EXTENSION OF X DEFINED ON β

$u(t) = x_0 + \int_0^{\beta} f(u(s))ds$

SO NOW ODE IS DEFINED ON $[0, \beta]$

NOW, $\begin{cases} v' = f(v) \\ v(\beta) = u(\beta) = x_1 \end{cases}$ $v(t)$ $\beta - a < t < \beta + a$

$\Rightarrow \beta = \infty$ $\Downarrow$ ∎