Section 3.1 Week 2.

I. Some terms to clarify

(1) Connectives : and , or , if ... then

(2) Exclusive or : One or the other event can take place , not both.

(3) Inclusive or : one or the other , or both events, can take place.

II. Statement and Logical Connectives

Definition: A sentence that can be judged either true or false is called a statement. Labeling a statement true or false is called assigning a truth value to the statement.

Examples : (1) Today is Friday (False)

(2) God is everlasting (True)

Definitions: 1) A sentence that convey only one idea is a simple statement.

2) Statements consisting of two or more simple statements are called compound statements. The connectives often used to join two simple statements are : and, or, if ... then... , if and only if .

To reduce the amount of writing, we usually represent a statement with a letter, often p , q , r , or s.

III. Quantifiers.

The words all , none (or no) and some are called quantifiers.

IV. And Statements

The conjunction is symbolized by $\wedge$ (read "and")

Examples:

(1) Let p represents the statement " He is good at math"

q represents the statement " He is good at sports"

then $p \wedge q$ represents " He is good at math and he is good at sports"

(2) Let p represents the statement " He studies hard"

q represents the statement " He sleeps a lot"

then $p \wedge q$ rep. " He studies hard and he sleeps a lot."

V. Or statements

The disjunction is symbolized by $\vee$ (read or)

Examples : (1) p: He will take MA 115

q: He will take BIO 201

$p \vee q$: He will take MA 115 or he will take BIO 201

N.B. : $p \vee q$ means he will take MA 115 or BIO 201 or both. We're using inclusive or.

VI. Grouping of simple statements.

When a compound statement contains more than one connective, a comma can be used to indicate which simple statements are to be grouped together.

When we write the compound statement symbolically, the statements on the same side of the comma are to be grouped together within parentheses.

Example :

Let

p: Dinner includes soup

q: Dinner includes salad

r: Dinner includes the vegetable of the day.

	Statement	Symbolic rep.	Type of statement
a)	Dinner includes soup, and salad or the vege. of the day	$p \wedge (q \vee r)$	conjunction
b)	Dinner includes soup and salad, or the vege. of the day.	$(p \wedge q) \vee r$	disjunction.

VII. If - Then statements

The conditional is symbolized by $\rightarrow$. (read "if - then") The statement $p \rightarrow q$ is read "If p, then q" The part before the arrow is called antecedent, and the part follows the arrow is called consequent. In $p \rightarrow q$, p is the antecedent and q is the consequent.

Examples:

(1) p: You get 3 A's
q: I will buy you a car.
$p \rightarrow q$: If you get 3A's, I will buy you a car.

(2) p: You work hard.
q: You will get promoted
$p \rightarrow q$: If you work hard, then you will get promoted

VIII. If and only if statements

The biconditional is symbolized by $\leftrightarrow$ (read if and only if)
$p \leftrightarrow q$ is read "p if and only if q"

Example:

p: you work hard
q: you will get promoted
$p \leftrightarrow q$: you work hard iff you will get promoted

IX. Negation

Definition: Negation is used to change a statement to its opp. meaning.

N.B:
- The negation of a true statement is always a false statement.
- The negation of a false statement is always a true statement
- Negation is symbolized by $\sim$ (read "not")
- A negation symbol has the effect of negating only the statement that directly follows it

Example: p : It is raining
$\sim p$: It is not raining

A. Negation of Quantifiers

Form of statement	Form of negation
All are	Some are not
None are	Some are / At least one is
Some are / At least one is	None are
Some are not	All are

Examples: 1) p: All monks are bald
$\sim p$: Some monks are not bald
2) p: Not one girl accepts a date from him.
$\sim p$: Some girls accept a date from him.

B. Negation in compound statements

Examples: p: Maria will go to the circus
q : Maria will go to the zoo.

(a) $p \vee \sim q$: Maria will go to the circus or she will not go to the zoo

(b) $\sim p \wedge q$: Maria will not go to the circus and she will go to the zoo.

(c) $\sim p \vee \sim q$: Maria will not go to the circus or she will not go to the zoo.

(d) $p \rightarrow \sim q$: If Maria goes to the circus, then she will not go to the zoo.

(e) $\sim p \leftrightarrow q$: Maria will not go to the circus iff she goes to the zoo.

C. Negation of Compound Statements

p: I will buy you a car.

q: You will pay the insurance.

(a) $\sim(p \wedge q)$: It is false that I will buy you a car and you will pay the insurance.

(b) $\sim p \vee \sim q$: I will not buy you a car or you will not pay the insurance.

will see later that $\sim(p \wedge q)$ is equivalent to $\sim p \vee \sim q$.

(c) $\sim(p \rightarrow q)$: It is false that if I buy you a car, then you will pay the insurance.