Section 2.3 Week 1.

I. Definition

1.) Disjoint sets : Two sets are disjoint when they have no elements in common.

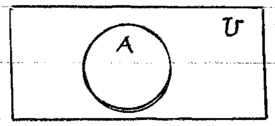
ex : The set of even number & the set of odd number are disjoint.

2.) Overlapping sets : Two sets are overlapping when they have elements in common.

II. Venn Diagram (by John Venn 1834-1923)

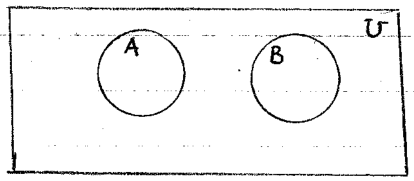
A rectangle representing the universal set.

The subsets are usually represented by circles.



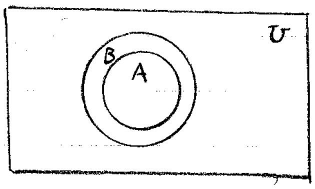
III. Different cases

- Case 1 : Disjoint sets



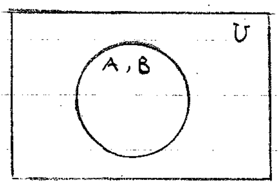
A & B are disjoint.

- Case 2 : Subsets



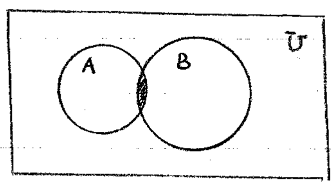
$A \subseteq B$

- Case 3 : Equal sets



$A = B$

- Case 4 : Overlapping sets



A and B are overlapping sets

IV. Definitions

• Complement : The complement of set A, symbolized by A', is the set of all the elements in the universal set that are not in set A.

Example :

1) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7\}$

$A' = \{2, 4, 6, 8, 9, 10\}$

2) $U = \{\alpha, \beta, \gamma, \sigma, \delta, \omega\}$

$A = \{\alpha, \beta\}$

$A' = \{\gamma, \sigma, \delta, \omega\}$

• Intersection : The intersection of sets A and B, symbolized by $A \cap B$, is the set containing all the elements that are common to both set A and set B

Example :

1) $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{1, 3, 7, 9, 10, 12\}$

$A \cap B = \{1, 3\}$

2) $A = \{\Delta, \#, ?, @\}$

$B = \{!, \Delta, \square, ?, \#\}$

$A \cap B = \{\Delta, \#, ?\}$

3) $U = \{a, b, c, d, e, f, g\}$

$A = \{a, b, c, d\}$

$B = \{a, c, e, f\}$

$A' = \{e, f, g\}$

$A' \cap B = \{e, f\}$

$(A' \cap B)' = \{a, b, c, d, g\}$

• Union : The <u>union</u> of set A and set B, symbolized by $A \cup B$, is the set containing all the elements of set A or set B (or of both sets)

Example :

1) $A = \{\alpha, \beta, \gamma\}$

$B = \{\alpha, \sigma, \delta, \varepsilon\}$

$A \cup B = \{\alpha, \beta, \gamma, \sigma, \delta, \varepsilon\}$

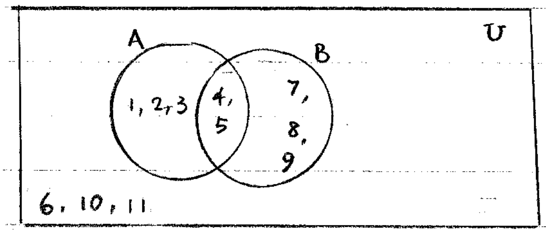
2) $A = \{\Delta, \square, \#\}$

$B = \{\Delta, ?, !\}$

$A \cup B = \{\Delta, \square, ?, \#, !\}$

<u>Using Venn Diagram</u>

1)



a) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$A = \{1, 2, 3, 4, 5\}$

$B = \{4, 5, 7, 8, 9\}$

$B' = \{1, 2, 3, 6, 10, 11\}$

$A \cap B' = \{1, 2, 3\}$

$A \cup B' = \{1, 2, 3, 4, 5, 6, 10, 11\}$

Further Example:

Union & Intersection

$U = \{a, b, c, d, e, f, g\}$

$A = \{a, b, e, g\}$

$B = \{a, c, d, e\}$

$C = \{b, e, f\}$

a) $(A \cup B) \cap (A \cup C)$

First, let's calculate $A \cup B$ & $A \cup C$

$A \cup B = \{a, b, c, d, e, g\}$

$A \cup C = \{a, b, e, f, g\}$

then $(A \cup B) \cap (A \cup C) = \{a, b, e, g\}$

(b) $(A \cup B) \cap C'$

First, calculate $A \cup B$ & C'

$A \cup B = \{a, b, c, d, e, g\}$

$C' = \{a, c, d, g\}$

Then $(A \cup B) \cap C' = \{a, c, d, g\}$

(c) $A' \cap B'$

First, write down A' & B'

$A' = \{c, d, f\}$

$B' = \{b, f, g\}$

$A' \cap B' = \{f\}$

V. The meaning of And or Or.

<u>And</u> is generally interpreted to mean intersection

<u>Or</u> is generally interpreted to mean union

VI. The Relationship between $n(A \cup B)$, $n(A)$, $n(B)$ & $n(A \cap B)$

For any finite sets A and B

$$n(A \cup B) = n(A) + n(B) - n(A \cup B)$$

Example:

The results of a survey of students showed that 400 of them have a laptop, 550 have a desktop, and 70 of them have both How many students have a laptop of desktop?

Let A = set of students who have a laptop

B = set of students who have a desktop

then $n(A) = 400$, $n(B) = 550$

$n(A \cap B) = 70$

$\therefore n(A \cup B) = 400 + 550 - 70 = 880$.

VII. Definition

The difference of two sets A and B, symbolized $A - B$, is the set of elements that belong to set A but not to set B.

Example:

$U = \{a, b, c, d, e, f, g, h, i, j, k\}$

$A = \{b, c, e, f, g, h\}$

$B = \{a, b, c, g, i\}$

$C = \{b, e, g\}$

- $A - B = \{e, f, h\}$
- $A - C = \{c, f, h\}$
- $A' = \{a, d, i, j, k\}$

  $A' - B = \{d, j, k\}$
- $C' = \{a, c, d, f, h, i, j, k\}$

  $A - C' = \{b, e, g\}$

VIII. Cartesian Product

The cartesian product of set A and set B, symbolized by $A \times B$ (read A cross B) is the set of all possible ordered pairs of the form (a,b), where $a \in A$, $b \in B$.

Example: $A = \{a, b, c, d\}$

$B = \{1, 2, 3\}$

$$A \times B = \{(a,1), (b,1), (c,1), (d,1) \\ (a,2), (b,2), (c,2), (d,2) \\ (a,3), (b,3), (c,3), (d,3)\}$$

$$B \times A = \{(1,a), (1,b), (1,c), (1,d) \\ (2,a), (2,b), (2,c), (2,d) \\ (3,a), (3,b), (3,c), (3,d)\}$$

N.B. $A \times B$ is not the same as $B \times A$.

$$A \times A = \{(a,a), (a,b), (a,c), (a,d) \\ (b,a), (b,b), (b,c), (b,d) \\ (c,a), (c,b), (c,c), (c,d) \\ (d,a), (d,b), (d,c), (d,d)\}$$

$$B \times B = \{(1,1), (1,2), (1,3) \\ (2,1), (2,2), (2,3) \\ (3,1), (3,2), (3,3)\}$$

If $n(A) = m$, $n(B) = n$, then $n(A \times B) = m \times n$