Section 3.4 Equivalent Statement Week 3

I Defn: Two statements are equivalent, symbolized $\Leftrightarrow$, if both statements have exactly the same truth values in the answer columns of the truth tables.

Example: Are $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$?

construct the truth table:

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T T T	T T T
T	T	F	T T T	T T F
T	F	T	T T T	F T T
T	F	F	T F F	F F F
F	T	T	F F T	F F T
F	T	F	F F T	F F F
F	F	T	F F T	F F F
F	F	F	F F F	F F F
			①	②

since ① & ② are the same $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$.

Example:

Determine which statement is eq to "It is not true that the tire is both out of balance and flat"

a) If the tire is not flat, then the tire is not out of balance.

b) The tire is not out of balance or the tire is not flat.

c) The tire is not flat and the tire is not out of balance

d) If the tire is not out of balance, then the tire is not flat

Let p: The tire is out of balance.

q: The tire is flat

so we want to find the eq. statement to $\sim(p \wedge q)$

In symbol a) $\sim q \rightarrow \sim p$ c) $\sim q \wedge \sim p$

b) $\sim p \vee \sim q$ d) $\sim p \rightarrow \sim q$

Now let's write down the truth table:

p	q	$\sim(p \wedge q)$	$\sim q \rightarrow \sim p$	$\sim p \vee \sim q$	$\sim q \wedge \sim p$	$\sim p \rightarrow \sim q$
T	T	F T	F T F	F F F	F F F	F T F
T	F	T F	T F F	F T T	T F F	F T T
F	T	T F	F T T	T T F	F F T	T F F
F	F	T F	T T T	T T T	T T T	T T T
		*	a	b	c	d

$\therefore$ b) is eq. to the statement

ie $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$

II. De Morgan's Laws

De Morgan's Laws:

1) $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$
2) $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

Proof:

1)

p	q	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F T	F F F
T	F	T F	F T T
F	T	T F	T T F
F	F	T F	T T T
		1	2

$\therefore$ $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$

2)

p	q	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F T	F F F
T	F	F T	F F T
F	T	F T	T F F
F	F	T F	T T T
		1	2

$\therefore$ $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$.

Example: Use De Morgan's Law to write an eq. statement to
" It is not true that tomatoes are poisonous or eating peppers cures the common cold "

Let p: Tomatoes are poisonous.
q: Eating peppers cures the common cold

So the statement is $\sim(p \vee q)$
using De Morgan's Law $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$
$\therefore$ An eq. statement is
Tomatoes are not poisonous & eating peppers does not cure the common cold.

III. Equivalence to conditional statement

$$p \to q \Leftrightarrow \sim p \vee q$$

Proof:

p	q	$p \to q$	$\sim p$	$\vee$	q
T	T	T	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	F
		①		②	

$\therefore \quad p \to q \Leftrightarrow \sim p \vee q$.

IV. Negation of the Conditional Statement

We know from III that
$$p \to q \Leftrightarrow \sim p \vee q$$
then $\sim(p \to q) \Leftrightarrow \sim(\sim p \vee q)$
thus $\sim(p \to q) \Leftrightarrow p \wedge \sim q$ using De Morgan's Law.
$\therefore \sim(p \to q) \Leftrightarrow p \wedge \sim q$.

Example: Write an eq. statement to " It is false that if it is snowing then we cannot go to the basketball game "

Let p: It is snowing

q: We go to the basketball game

the statement in symbol is $\sim(p \to \sim q)$

since $\sim(p \to \sim q) \Leftrightarrow p \wedge q$,

an eq. statement is :

It is snowing and we go to the basketball game.

V. Variations of the conditional statement

There are 3 variations of the conditional statement, summarized below:

Name	Symbolic form	Read
Conditional	$p \to q$	If p, then q.
Converse of cond.	$q \to p$	If q, then p.
Inverse of cond.	$\sim p \to \sim q$	If not p, then not q.
Contrapositive of cond.	$\sim q \to \sim p$	If not q, then not p.

Let's see which ones are eq.

p	q	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

$\therefore$ $p \to q \Leftrightarrow \sim q \to \sim p$

$q \to p \Leftrightarrow \sim p \to \sim q$.

Example: Write the contrapositive of the statement

If two lines do not intersect in at least one point, then the two lines are parallel.

Let p: Two lines intersect in at least one ~~pt~~

q: Two lines are parallel.

So the original statement is $\sim p \to q$, which is $\Leftrightarrow$ to $\sim q \to p$.

ie If two lines are not parallel, then they intersect in at least one point.

Example: Determine which, if any, of the three statements are eq.

a) The office is not cool and the copier is jammed.

b) If the office is not cool, then the copier is not jammed.

c) It is false that the office is cool or the copier is not jammed.

Let p: The office is cool

q: The copier is jammed.

a) $\sim p \wedge q$

b) $\sim p \to \sim q$

c) $\sim(p \vee \sim q) \Leftrightarrow \sim p \wedge q = q \wedge \sim p$

$\therefore$ a) & c) are eq.