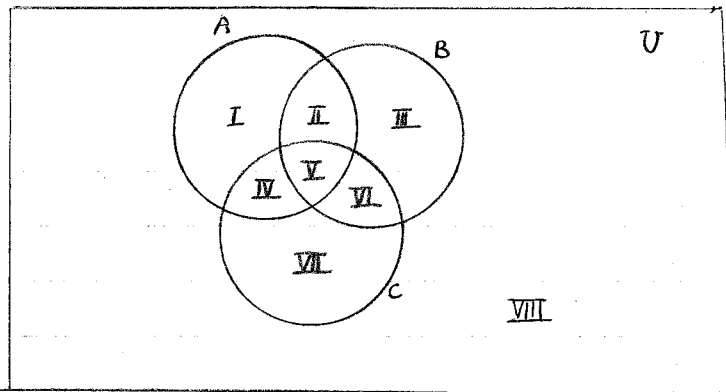I. General procedure for constructing Venn Diagram with three sets.

1. Determine the elements to be placed in region V by finding the elements that are common to all three sets, $A \cap B \cap C$.

2. Determine the elements to be placed in region II. Find the elements in $A \cap B$. The elements in this set belong to regions II and V. Place the elements in the set $A \cap B$ that are not listed in region V in region II. The elements in region IV and VI are found in similar manner.

3. Determine the elements to be placed in region I by determining the elements in set A that are not in region II, IV & V. The elements in regions III & VII are found in a similar manner.

4. Determine the elements to be placed in region VIII by finding the elements in the universal set that are not in regions I thr. VII.
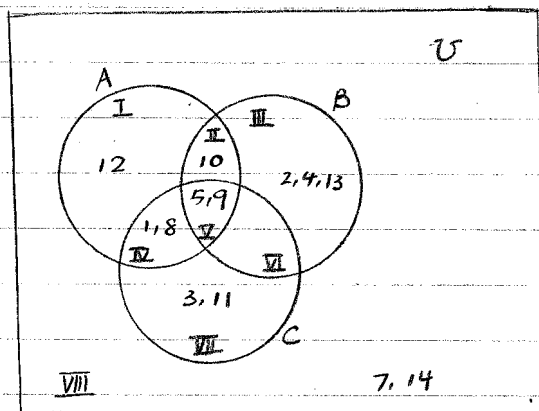
(Book p73)

II. Example

1.) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

$A = \{1, 5, 8, 9, 10, 12\}$

$B = \{2, 4, 5, 9, 10, 13\}$

$C = \{1, 3, 5, 8, 9, 11\}$

- First, find $A \cap B \cap C$

  $A \cap B \cap C = \{5, 9\}$

- Then, place 5, 9 in region V

- Next, find $A \cap B$

  $A \cap B = \{5, 9, 10\}$

- Then, place 10 in region II.

- Now, find $A \cap C$

  $A \cap C = \{1, 5, 8, 9\}$

- Then, place 1, 8 in region IV

- Next, find $B \cap C$

  $B \cap C = \{5, 9\}$

- 5, 9 are already in V, ∴ there is no element in VI

- To find elements in region I, find $A - A \cap B - A \cap C$.

  $A - A \cap B - A \cap C = \{1, 5, 8, 9, 10, 12\} - \{5, 9, 10\} - \{1, 5, 8, 9\}$

  $= \{12\}$

- Place 12 in region I.

- To find elements in region III, find $B - A \cap B - B \cap C$

  $B - A \cap B - B \cap C = \{2, 4, 5, 9, 10, 13\} - \{5, 9, 10\} - \{5, 9\}$

  $= \{2, 4, 13\}$

- Place 2, 4, 13 in region III

- To find elements in region VII, find $C - B \cap C - A \cap C$

  $C - B \cap C - A \cap C = \{1, 3, 5, 8, 9, 11\} - \{5, 9\} - \{1, 5, 8, 9\}$

  $= \{3, 11\}$

- Place 3, 11 in region VII.

- To find elements in region VIII, find $U - A - B - C$

  $U - A - B - C = \{7, 14\}$

- Place 7, 14 in region VIII.

III. Verification of Equality of Sets.

For clarity, we refer to operations on sets as statements.

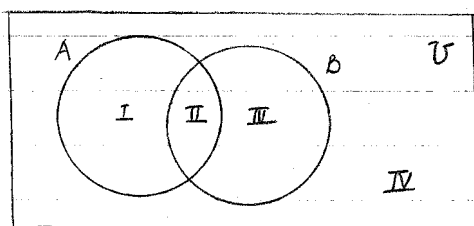Two statements are equal if they are represented by the same regions in the Venn diagram.

IV. Examples

1) Determine whether $(A \cap B)' = A' \cup B'$ $\forall$ sets A & B.

Find $(A \cap B)'$

set	corresponding regions
A	I, II
B	II, III
A∩B	II
(A∩B)'	I, III, IV

Find $A' \cup B'$

set	corresponding regions
A'	III, IV
B'	I, IV
A'∪B'	I, III, IV

Since both statements are represented by the same regions, I, III, IV of the Venn diagram, $(A \cap B)' = A' \cup B'$ $\forall$ sets A & B.

2) Determine whether $A \cap B = A' \cup B$ $\forall$ sets A & B.

Find $A \cap B$

set	corresponding regions
A	I, II
B	II, III
A∩B	II

Find $A' \cup B$

set	corresponding regions
A'	III, IV
B	II, III
A'∪B	II, III, IV

Since the statements are represented by different regions in the Venn diagram, the statements are not equal in general.

14.

I. De Morgan's Laws.

     1.) $(A \cap B)' = A' \cup B'$

     2) $(A \cup B)' = A' \cap B'$

               $\forall$ sets A and B.

1) is proved on p14

Let's prove 2)

Find $(A \cup B)'$			Find $A' \cap B'$	
set	corresponding regions		set	corresponding regions
A	I, II		A'	III, IV
B	II, III		B'	I, IV
A ∪ B	I, II, III		A' ∩ B'	IV
(A ∪ B)'	IV			

Since both statements are represented by the same region IV, on the Venn diagram, they are equal.
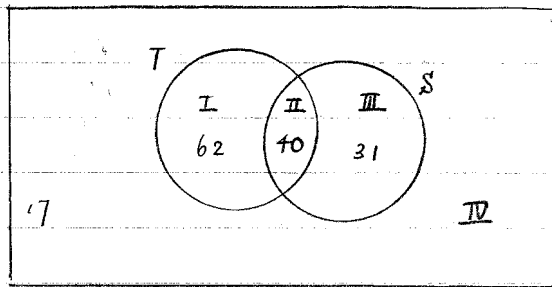
I. Examples

1) 150 members of a health club were asked which equipment they had used
    in the previous month. The result is as follow:

        102 used the treadmills

        71 used the Stair Masters.

        40 used both types.

Region	# of elements
I	$102 - 40 = 62$
start here → II	40
III	$71 - 40 = 31$
IV	$150 - 62 - 40 - 31 = 17$

a) How many member use treadmills only?
    ie how many elements are in region I?
    Ans: 62

b) How many member use Stair Masters only?
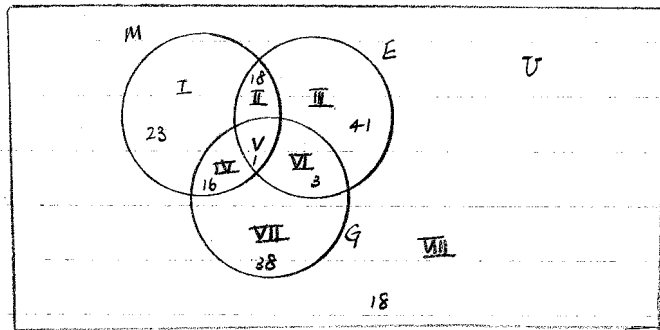   ie how many elements are in region III?
    Ans: 31

c) How many member do not use either?
    ie how many elements are in region IV?
    Ans: 17

2) A man interviewed 148 people in a shopping mall to find out some of their cooking methods:

58 use microwave ovens

63 use electric ranges

58 use gas ranges

19 use microwave ovens & electric ranges

17 use microwave ovens & gas ranges

4 use both gas & electric ranges

1 uses all three.

Region	# of elements
I	$58 - 18 - 1 - 16 = 23$
II	$19 - 1 = 18$
III	$63 - 18 - 1 - 3 = 41$
IV	$17 - 1 = 16$
V	1
VI	$4 - 1 = 3$
VII	$58 - 16 - 1 - 3 = 38$
VIII	$148 - 23 - 18 - 41 - 16 - 1 - 3 - 28 = 8$

start here

a) How many people use electric ranges only ? 41

b) " " " " gas ranges only ? 38

c) " " " " neither ? 8 (region VIII)

d) " " " " M or E but not G ? 18 (region II)

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