PRINT NAME: ______________________________________.

Math 423:	Homework: Cyclic and Dihedral Groups Assignment

Show **your** work carefully. Use full sentences, proper grammar and be precise. You don't have to copy the problem statement again, but, your solution must be self-contained. 60pts to earn here.

Problem 25: Let G_1, G_2 be groups. Suppose $H_1 \leq G_1$ and $H_2 \leq G_2$. Prove $H_1 \times H_2$ is a subgroup of $G_1 \times G_2$.

Problem 26: Suppose a group G has an element $a \in G$ with order 30. Find **all** powers of a which have order 2, 3 and 5.

Problem 27: Which of the multiplicative groups $U(7), U(10), U(12), U(14)$ are isomorphic ?

Problem 28: Let G be any finite group with no proper, nontrivial subgroups, and assume $|G| > 1$. Prove that G must be isomorphic to $\mathbb{Z}_p$ for some prime p.

Problem 29: Prove that $\text{aut}(\mathbb{Z}_n) \cong U(n)$. Here $\text{aut}(\mathbb{Z}_n)$ denotes the set of all isomorphism from $\mathbb{Z}_n$ to $\mathbb{Z}_n$ and $U(n)$ is the multiplicative group of units in $\mathbb{Z}_n$.

Problem 30: Find the order of each element in D_4 and write the Cayley table for D_4

Problem 31: Let $H = \left\{ \pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \pm \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \pm \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \pm \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \right\} \subseteq \text{GL}_2(\mathbb{C})$. Prove D_4 is isomorphic to H.

Problem 32: Show that D_n is isomorphic to a subgroup of S_n for $n \geq 3$.

Problem 33: What is the largest order of an element in $S_5 \times S_8$?

Problem 34: Chapter 4, Exercise # 10 (generators in cyclic group)

Problem 35: $\mathbb{Z}_{10} \times \mathbb{Z}_9$ is a group of order 90.

(a.) Consider $\langle (2,3) \rangle \leq \mathbb{Z}_{10} \times \mathbb{Z}_9$. Find the order of the cyclic subgroup $\langle (2,3) \rangle$ and list all generators of this subgroup.

(b.) Is $\mathbb{Z}_{10} \times \mathbb{Z}_9$ a cyclic group ? If it is then how many distinct generators does $\mathbb{Z}_{10} \times \mathbb{Z}_9$ possess ?

Problem 36: Suppose D_8 has reflection y with $y^2 = 1$ and rotation x with order 8. Simplify the expression $x^3yx^2yxyx^5$ to an expression of the form y^ax^b for appropriate non-negative integers a, b.