PRINT NAME: ____________________________________.

MATH 423: HOMEWORK: HOMOMORPHISM AND REPRESENTATIONS ASSIGNMENT

Show **your** work carefully. Use full sentences, proper grammar and be precise. You don't have to copy the problem statement again, but, your solution must be self-contained. 60pts to earn here. I strongly recommend you finish and understand this assignment before attempting the Midterm.

Problem 37: Chapter 6, Exercise # 5 (groups isomorphic)
Remark: *note, one way to prove isomorphism is to write out both Cayley tables in such a way that the pattern of the table matches. Note, you have to order the elements to match the isomorphism in order that the pattern be seen, but that's easy to do in this problem.*

Problem 38: Chapter 6, Exercise # 7 (groups not isomorphic)

Problem 39: Chapter 6, Exercise # 37 (complex conjugation)

Problem 40: Chapter 6, Exercise # 41 (matrix model of complex numbers)

Problem 41: Chapter 6, Exercise # 47 (question about inner automorphism)

Problem 42: Chapter 10, Exercise # 56 (homorphism and its kernel)

Problem 43: State the theorems and definitions given in the video concerning homomorphisms in this week's assigned video viewing.

Problem 44: Suppose $\phi : G_1 \to G_2$ is a surjective group homomorphism and K is a subgroup of the G_2. Prove $\phi^{-1}(K) = \{g \in G_1 \mid \phi(g) \in K\}$ is a subgroup of G_1.

Problem 45: Use generalized cycle notation for the Klein 4-group $K = \{1, a, b, ab\}$ to help identify an explicit subgroup of S_4 to which K is isomorphic. This illustrates Cayley's Theorem.

Problem 46: Use generalized cycle notation for the quarternion 8-group $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$ to find an explicit subgroup of S_8 to which Q_8 is isomorphic. *I believe there is a typo in the video on Cayley's Theorem from this week around the 28:55 mark*

Problem 47: Suppose H is a subgroup of a cyclic group G. How many generators of H can you expect and how are these generators related to the generators of G ? It would be good for you to answer this in general, but at least answer the question in the following scenarios:

(a.) If $G = \mathbb{Z}_{100}$ and $H = \langle 10 \rangle$. Find all generators of H.

(b.) If $G = \langle a \rangle$ where $|a| = 45$. Find all generators of $H = \langle a^3 \rangle$.

Problem 48: Consider a group G with $x, y \in G$ and $x \neq y$ where $|x| = |y| = 2$. Explain why G is not a cyclic group.