

## MATH 423: HOMEWORK WEEK 5: QUOTIENT AND PRODUCT GROUPS ASSIGNMENT

Show **your** work carefully. Use full sentences, proper grammar and be precise. You don't have to copy the problem statement again, but, your solution must be self-contained. 60pts to earn here. By my count, at least 3 of these problems are solved in the videos assigned this week.

**Problem 49:** Chapter 7, Exercise # 14 (Lagrange's Theorem problem)

**Problem 50:** Let  $H$  be a subgroup of  $G$  and  $a, b \in G$ . Prove  $aH = bH$  if and only if  $a^{-1}b \in H$ .

**Problem 51:** Let  $G$  be a group of order  $p$  where  $p$  is prime. Prove  $G$  is cyclic.

**Problem 52:** Let  $G = \mathbb{Z}_3 \times \mathbb{Z}_6$ . If  $H = \langle (1, 2) \rangle$  and  $K = \langle (1, 3) \rangle$  then find the addition table for  $G/H$  and  $G/K$ .

**Problem 53:** Let  $G$  be a group. Prove: If  $Z(G) \trianglelefteq G$ .

**Problem 54:** Find the center of  $D_4$  and construct the Cayley table of the factor group  $D_4/Z(D_4)$ . To which well-known group is this factor group isomorphic ?

**Remark:** *you should find that the center of  $D_4$  has two elements. I recommend you calculate using the generators and relations formulation of  $D_4$  given by  $x^4 = 1$  and  $y^2 = 1$  where  $xyx = x^{-1} = x^3$ .*

**Problem 55:** Let  $D_3 = \{1, x, x^2, y, xy, x^2y\}$  denote a dihedral group.

(a.) Is  $H = \langle x \rangle$  a normal subgroup ? If so, what  $\mathbb{Z}_n$  is isomorphic to  $D_3/H$  ?

(b.) Is  $K = \langle y \rangle$  a normal subgroup ? If so, what  $\mathbb{Z}_n$  is isomorphic to  $D_3/K$  ?

(c.) Is  $H \times K \cong D_3$  ?

**Problem 56:** List the non-isomorphic abelian groups of order 56. Circle any that are cyclic.

**Remark:** *there are only three cases here.*

**Problem 57:** Chapter 8, Exercise # 37 (nonzero complex numbers under multiplication are not isomorphic to direct product of nonzero real numbers under multiplication)

**Problem 58:** Chapter 9, Exercise # 11 (factor group of cyclic group is cyclic)

**Problem 59:** Chapter 10, Exercise # 8 (alternating group is normal)

**Problem 60:** Chapter 10, Exercise # 11 (isomorphism question, best solution is to construct appropriate homomorphism as to apply first isomorphism theorem)