

MATH 423: HOMEWORK: QUOTIENT AND PRODUCT GROUPS ASSIGNMENT

Show **your** work carefully. Use full sentences, proper grammar and be precise. You don't have to copy the problem statement again, but, your solution must be self-contained. 60pts to earn here. By my count, at least 3 of these problems are solved in the videos assigned this week.

Problem 49: Chapter 7, Exercise # 16 (Lagrange's Theorem problem)

Problem 50: Let H be a subgroup of G and $a, b \in G$. Prove $aH = bH$ if and only if $a^{-1}b \in H$.

Problem 51: Let G be a group of order p where p is prime. Prove G is cyclic.

Problem 52: Let $G = \mathbb{Z}_3 \times \mathbb{Z}_6$. If $H = \langle (1, 2) \rangle$ and $K = \langle (1, 3) \rangle$ then find the addition table for G/H and G/K .

Problem 53: Let G be a group. Prove: If $Z(G) \trianglelefteq G$.

Problem 54: Find the center of D_4 and construct the Cayley table of the factor group $D_4/Z(D_4)$. To which well-known group is this factor group isomorphic ?

Remark: *you should find that the center of D_4 has two elements. I recommend you calculate using the generators and relations formulation of D_4 given by $x^4 = 1$ and $y^2 = 1$ where $xyx = x^{-1} = x^3$.*

Problem 55: Let $D_3 = \{1, x, x^2, y, xy, x^2y\}$ denote a dihedral group.

(a.) Is $H = \langle x \rangle$ a normal subgroup ? If so, what \mathbb{Z}_n is isomorphic to D_3/H ?

(b.) Is $K = \langle y \rangle$ a normal subgroup ? If so, what \mathbb{Z}_n is isomorphic to D_3/K ?

(c.) Is $H \times K \cong D_3$?

Problem 56: List the non-isomorphic abelian groups of order 56. Circle any that are cyclic.

Remark: *there are only three cases here.*

Problem 57: Chapter 8, Exercise # 35 (nonzero complex numbers under multiplication are not isomorphic to direct product of nonzero real numbers under multiplication)

Problem 58: Chapter 9, Exercise # 11 (factor group of cyclic group is cyclic)

Problem 59: Chapter 10, Exercise # 8 (alternating group is normal)

Problem 60: Chapter 10, Exercise # 11 (isomorphism question, best solution is to construct appropriate homomorphism as to apply first isomorphism theorem)