PRINT NAME: ____.

MATH 423: HOMEWORK: RINGS AND IDEALS ASSIGNMENT

Show **your** work carefully. Use full sentences, proper grammar and be precise. You don't have to copy the problem statement again, but, your solution must be self-contained. 60pts to earn here.

Problem 61: Chapter 12, Exercise # 40 (is this a subring ?)

Problem 62: Chapter 13, Exercise # 30 (show set is field)

Problem 63: Chapter 14, Exercise # 41 (calculate inverse in quotient ring)

Problem 64: Chapter 15, Exercise # 14 (ring isomorphism)

Problem 65: Suppose R is a ring with $x^2 = x$ for each $x \in R$. Show that R is a commutative ring.

Problem 66: An idempotent of a ring is an element e such that $e^2 = e$. Prove that the only idempotents in an integral domain are 0 and 1 where 1 denotes the multiplicative identity of the ring.

Problem 67: Suppose I is any set and let R be the collection of all subsets of I. Define addition and multiplication of subsets $A, B \subseteq I$ as follows:

$$A + B = A \cup B \qquad \& \qquad A \cdot B = A \cap B$$

Is R a commutative ring under this addition and multiplication ?

Problem 68: Let $R = \mathbb{Z}_4[i] = \{a + ib \mid a, b \in \mathbb{Z}_4,\ i^2 = -1\}$ where addition and multiplication are defined in the expected manner. Find the group of units for R.

Problem 69: Let R be a commutative ring with identity 1. Define $\langle a \rangle = \{ra \mid r \in R\}$. Prove $\langle a \rangle$ is an ideal of R.

Problem 70: Explain how we can construct the complex number system using a quotient space of polynomials.

Problem 71: Find each maximal ideal I of $R = \mathbb{Z}_8 \times \mathbb{Z}_{30}$. Explain both why your choices are reasonable and why there are no other choices beyond your list.

Problem 72: Suppose a commutative ring R has an element a such that $a^n = 0$ for some positive integer n. Also suppose u is a unit in R. Prove $u - a$ is a unit in R.
Hint: think about the geometric series.