PRINT NAME: ________________________________.

Math 423:	Homework: Polynomials Assignment

Show **your** work carefully. Use full sentences, proper grammar and be precise. You don't have to copy the problem statement again, but, your solution must be self-contained. 60pts to earn here.

Problem 73: Once more let $R = \left\{ \begin{bmatrix} x & y \\ y & x \end{bmatrix} \;\middle|\; x, y \in \mathbb{Z} \right\}$. Define $\varphi : R \to \mathbb{Z}$ by $\varphi(X) = X_{11} - X_{12}$ for each $X \in R$ and complete the following:

(a.) Show φ is a ring homomorphism.

(b.) Prove φ is a surjection.

(c.) Describe $\mathrm{Ker}(\varphi)$.

(d.) Find a known ring to which $R/\mathrm{Ker}(\varphi)$ is isomorphic.

(e.) Is $\mathrm{Ker}(\varphi)$ a prime ideal?

(f.) Is $\mathrm{Ker}(\varphi)$ a maximal ideal?

Problem 74: Chapter 16, Exercise # 2 (polynomial expression vs. function)

Problem 75: Chapter 17, Exercise # 18 (irreducible polynomial)

Problem 76: Chapter 17, Exercise # 19 (factoring)

Problem 77: Chapter 17, Exercise # 33 (interplay between maximal ideals and irreducibility)

Problem 78: Chapter 18, Exercise # 6 (associates)

Problem 79: Find the multiplicative inverse of the given elements in the given fields:

(a.) $[a + bx]$ in $\mathbb{Q}[x]/\langle x^2 - 2\rangle$

(b.) $[x^2 - 2x + 1]$ in $\mathbb{Q}[x]/\langle x^3 - 2\rangle$

(c.) $[x]$ in $\mathbb{Z}_5[x]/\langle x^3 + x + 1\rangle$

Problem 80: For which values of $a = 1, 2, 3, 4$ is $\mathbb{Z}_5[x]/\langle x^2 + a\rangle$ a field ?

Problem 81: Prove $1 + \sqrt{-3}$ is an irreducible in $\mathbb{Z}[\sqrt{-3}]$. *hint: see Video 27.*

Problem 82: Find the minimal polynomial for $\alpha \in E$ over F for:

(a.) $\alpha = 3 + i$ over $F = \mathbb{R}$, $E = \mathbb{C}$

(b.) $\alpha = \sqrt{2} - \sqrt{3}$ over $F = \mathbb{Q}$, $E = \mathbb{R}$.

for part (a.) I would like you to explain how you know the polynomial you find is irreducible. Proof of irreducibility of the polynomial for part (b.) is more involved and I don't require that in the solution this week.

Problem 83: Find all the irreducible monic quadratic polynomials over $\mathbb{Z}_3$. Hint: $x^2 + bx + c$ with $b, c \in \mathbb{Z}_3$ gives just nine choices to consider.

Problem 84: Prove $\mathbb{R}[x]/\langle x - a\rangle$ is isomorphic to $\mathbb{R}$ as a ring. Hint: study $\Psi(f(x) + \langle x - a\rangle) = f(a)$.