PRINT NAME: ____.

Math 423:	Homework: Fields Assignment

Show **your** work carefully. Use full sentences, proper grammar and be precise. You don't have to copy the problem statement again, but, your solution must be self-contained. 60pts to earn here.

Problem 85: Consider $E = \mathbb{Z}_7[x]/\langle x^4 + 3x^2 + x - 1\rangle$. Write $\alpha = x + \langle x^4 + 3x^2 + x - 1\rangle$ thus a typical element in $z \in E$ has the form $z = a + b\alpha + c\alpha^2 + d\alpha^3$ where $a, b, c, d \in \mathbb{Z}_7$. Find α^{-1}.

Problem 86: Suppose E is a finite extension field of F and suppose K is a finite extension field of E then K is a finite extension field of F. We denote the dimension of E over F as a vector space by $[E : F]$. Likewise, $[K : E]$ is the dimension of K as a vector space over E. Prove $[K : F] = [K : E][E : F]$. *hint: see video 30*

Problem 87: Chapter 20, Exercise # 7 (extension by elements of base field goes nowhere)

Problem 88: Chapter 20, Exercise # 29 (two rings to compare)

Problem 89: Chapter 20, Exercise # 31 (the answer should be in terms of β)

Problem 90: Chapter 21, Exercise # 3 (infinite algebraic extension)

Problem 91: Chapter 21, Exercise # 9 (think about using Problem 85 here)

Problem 92: Chapter 21, Exercise # 16 (you do not have to prove irreducibility of your minimal polynomial)

Problem 93: Chapter 17, Exercise # 14 (I might have a problem on the Final Exam which needs Eisenstein's Criteria to prove irreducibility)

Problem 94: Explain the relation between prime ideals and integral domains. What important theorem connects these concepts ?

Problem 95: Explain the relation between maximal ideals and fields. What important theorem connects these concepts ?

Problem 96: A typical quaternions has the form $t + ai + bj + ck$ where $t, a, b, c \in \mathbb{R}$ and i, j, k are imaginary units with the following algebraic rules:

$$ij = k,\ jk = i,\ ki = j,\ ji = -k,\ kj = -i,\ ik = -j,\ i^2 = j^2 = k^2 = -1$$

Why are the quaternions not a field ?