NAME ______________________________

MATH 101: FALL 2020 — QUIZ 2

You are allowed one page of notes and a calculator. No phones. More than 25pts to earn. $\boxed{Box}$ *your answers for full credit and show work. Thanks!*

Problem 1: (4pts) Solve $\begin{pmatrix} 2x+y=9 \\ 3x-y=-4 \end{pmatrix}$.

$+$

$5X = 5 \Rightarrow \boxed{X=1}$

Then $y = 9-2X = 9-2(1) = 7 \therefore \boxed{y=7}$

Problem 2: (4pts) Solve $\begin{array}{l} 3x+4y=5 \\ 5x+6y=9 \end{array}$.

$\Rightarrow 5(3X+4y) = 5(5)$

$\Rightarrow 3(5X+6y) = 3(9)$

Hence,

$$-\begin{pmatrix} 15X + 20y = 25 \\ 15X + 18y = 27 \end{pmatrix}$$

$2y = -2 \Rightarrow \boxed{y=-1}$

Then, $\underbrace{3X + 4(-1) = 5}$

$3X = 5+4 = 9$

$3X = 9$

$\boxed{X=3}$

Problem 3: (2pts) Determine if $(1,2)$ is a solution of the system of equations: $\begin{array}{l} x+y=3 \\ x-y=2 \end{array}$

Well, $X=1, y=2$ means $X+y = 1+2 = 3$ (good)

But, $X-y = 1-2 = -1 \neq 2$ (bad). Thus, No $(1,2)$ is not a sol^{n}.

Problem 4: (2pts) A fence is made such that its width is twice its length. In addition, the fence is constructed with 43 ft of fence. Find the length and width. *Hint: let the length be x and the width be y, find two equations and two unknowns which x and y must solve.*

$W = y = 2x$ ①

$l = x$

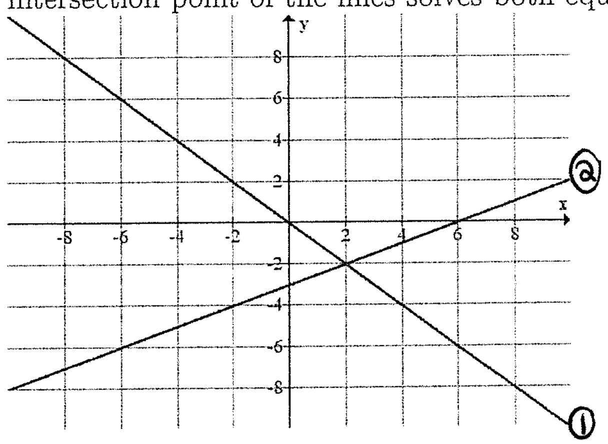
perimeter $= 2x + 2y = 43$ ②

Substitute ① into ② to obtain $2x + 2(2x) = 43 \Rightarrow 6x = 43$

thus $x = 43/6$ and $y = 2(43/6) = 43/3$

$$\boxed{\text{length} = 43/6 \text{ ft and width} = 43/3 \text{ ft}}$$

Problem 5: (3pts) Find two linear equations whose graphs are the lines given below. Also, verify the intersection point of the lines solves both equations.



① has y-intercept $b = 0$

Slope $= -1$ thus

$y = -x$ *

② has x-intercept of 6

thus $y = A(x-6)$

then $y = -6$ when $x = -6$

So $-6 = A(-6-6)$

$A = -6/-12 = 1/2$

$\therefore\ y = \frac{1}{2}(x-6)$ **

From * and ** I find,

$$\boxed{\begin{aligned} x + y &= 0 \\ x - 2y &= 6 \end{aligned}}$$

Is $(2,-2)$ a sol^{n}? (really THE sol^{n})

$2 + (-2) \overset{\checkmark}{=} 0$ and ~~2(-2)~~ $2 - 2(-2) \overset{\checkmark}{=} 6$. YEP.

Problem 6: (2pts) Let $P(x) = 2x^3 - 3x^2 + 7$. Calculate $P(2)$ and $P(-1)$.

$$P(2) = 2(2)^3 - 3(2)^2 + 7 = 16 - 12 + 7 = \boxed{11}$$

$$P(-1) = 2(-1)^3 - 3(-1)^2 + 7 = -2 - 3 + 7 = \boxed{2}$$

Problem 7: (7pts) Factor each polynomial below completely over $\mathbb{R}$,

(a.) $x^2 - 37 = x^2 - (\sqrt{37})^2 = \underline{(x - \sqrt{37})(x + \sqrt{37})}.$

(b.) $3x^2 + 6x + 3 = 3(x^2 + 2x + 1) = 3(x+1)(x+1) = \underline{3(x+1)^2}$

(c.) $2x^2 - 11x + 5 = \underline{(2x - 1)(x - 5)}.$

(d.) $x^3 + 4x^2 = \underline{x^2(x + 4)}.$

Remember: $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$, use $A = x$, $B = 3$

(e.) $x^3 - 27 = x^3 - 3^3 = \underline{(x-3)(x^2 + 3x + 9)}$

<u>Note</u>: $B^2 - 4AC = 9 - 4(1)(9) = -27 < 0$
so $x^2 + 3x + 9$ is irreducible.

(f.) $x^4 - 81 = (x^2)^2 - 9^2$
$= (x^2 - 9)(x^2 + 9)$
$= \underline{(x-3)(x+3)(x^2+9)}.$

(g.) $x^4 - 5x^2 + 4 = (x^2 - 1)(x^2 - 4)$
$= \underline{(x-1)(x+1)(x-2)(x+2)}.$

Problem 8: (2pts) Complete the square for $f(x) = x^2 + 6x - 20$ and factor $f(x)$ completely.

$$f(x) = x^2 + 6x - 20$$
$$= (x+3)^2 - 9 - 20$$
$$= (x+3)^2 - 29$$
$$= \underline{(x+3-\sqrt{29})(x+3+\sqrt{29})}.$$

Problem 9: (1pts) Solve $x^2 + 6x - 20 = 0$

$$(x+3-\sqrt{29})(x+3+\sqrt{29}) = 0$$

$$\Rightarrow \quad x+3-\sqrt{29} = 0 \quad \text{or} \quad x+3+\sqrt{29} = 0$$

$$\therefore \boxed{x = -3 \pm \sqrt{29}}$$

Problem 10: (3pts) The disciminant for $f(x) = ax^2 + bx + c$ is $b^2 - 4ac$. Calculate the discriminant for each $f(x)$ given below and factor $f(x)$ over $\mathbb{R}$ if possible.

(a.) $x^2 + 4x + 5$ $\quad a=1, \; b=4, \; c=5$

$b^2 - 4ac = 16 - 4(1)(5) = 16 - 20 = -4 < 0$

hence $x^2 + 4x + 5$ is irreducible over $\mathbb{R}$.

(b.) $x^2 + 10x - 13$ $\quad a=1, \; b=10, \; c=-13$

$b^2 - 4ac = 100 - 4(1)(-13) = 152 > 0 \; \therefore$ <u>can factor</u>

$$x^2 + 10x - 13 = (x+5)^2 - 25 - 13$$
$$= (x+5)^2 - 38 = \underline{(x+5-\sqrt{38})(x+5+\sqrt{38})}.$$

(c.) $x^2 - 6x + 9$ $= (x-3)(x-3)$

and $a=1, \; b=-6, \; c=9$

$b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0.$

yep. It was a repeated factor as expected.