NAME ______________________________

MATH 101: FALL 2020	TEST 2

You are allowed one page of notes and a calculator. No phones. More than 25pts to earn. $\boxed{\textit{Box}}$ *your answers for full credit and show work. Thanks!*

Problem 1: (15pts) Solve $-\begin{pmatrix} x + 2y = 5 \\ x - 5y = -9 \end{pmatrix}$

$2y - (-5y) = 5 - (-9)$

$7y = 14 \Rightarrow y = 14/7,$

$\boxed{y = 2}$

Then,

$x = 5 - 2y = 5 - 2(2) = 5 - 4 = 1 \quad \therefore \boxed{x = 1}$

We find sol^{n} of $\boxed{(1, 2)}$

Problem 2: (15pts) Solve $\begin{array}{l} 4x + 5y = 26 \quad ① \\ 6x + 7y = 38 \quad ② \end{array}$

Multiply ① by 3, $3(4x + 5y) = 3(26) = 78$

Multply ② by 2, $2(6x + 7y) = 2(38) = 76$

Then, we can cancel the 12x terms by subtracting,

$-\begin{pmatrix} 12x + 15y = 78 \\ 12x + 14y = 76 \end{pmatrix}$

$\boxed{y = 2}$

Then $4x + 5y = 4x + 10 = 26 \Rightarrow 4x = 16$

$\Rightarrow x = 16/4$

$\therefore \boxed{x = 4}$

Sol^{n} is, $\boxed{(4, 2)}$

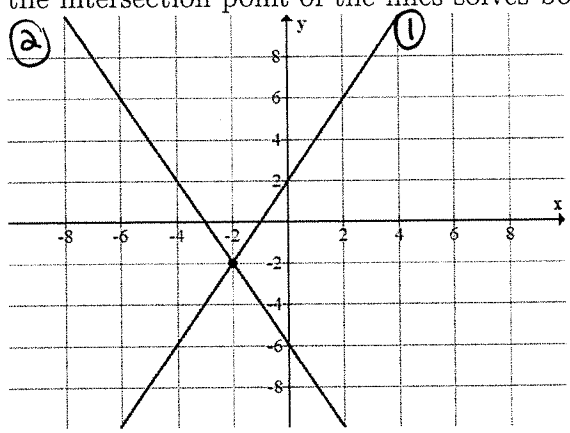
Problem 3: (15pts) Solve $\begin{array}{l} x + 2y = 3 \\ 2x + 4y = 4 \end{array}$.

$$\Rightarrow \begin{array}{r} (2x + 4y = 6) \\ -(2x + 4y = 4) \\ \hline 0 = 2 \end{array}$$

$\therefore$ No Sol^{n} exists.

Sol^{n} Set is empty.

Problem 4: (15pts) Find two linear equations whose graphs are the lines given below. Also, verify the intersection point of the lines solves both equations.



the point $(-2,-2)$ is on both lines.

Line 1 has y-intercept of 2.

Line 2 has y-intercept of -6

Line 1 has slope $m_1 = 2$

Line 2 has slope $m_2 = -2$

Thus, $\boxed{y = 2x+2 \ \& \ y = -2x-6}$

Or,

$$\boxed{\begin{array}{rl} 2x - y &= -2 \\ -2x - y &= 6 \end{array}}$$

We can check,

$2(-2) - (-2) = -4 + 2 \stackrel{\checkmark}{=} -2$

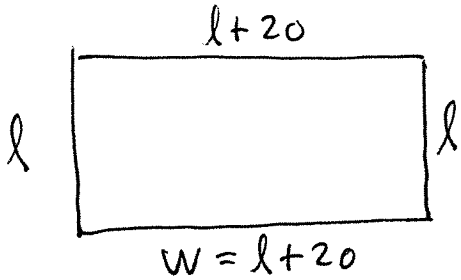
$-2(-2) - (-2) = 4 + 2 \stackrel{\checkmark}{=} 6$

Problem 5: (10pts) Let $P(x) = 3x^3 + 2x + 1$. Calculate $P(1)$ and $P(-1)$.

$P(1) = 3(1)^3 + 2(1) + 1 = 3 + 2 + 1 = \boxed{6}$

$P(-1) = 3(-1)^3 + 2(-1) + 1 = -3 - 2 + 1 = -5 + 1 = \boxed{-4}$

Problem 6: (10pts) A fence is made such that its width is 20 feet longer than its length. In addition, the fence is constructed with 65 ft of fence. Find the length and width.



$$l + l+20 + l + l+20 = 65$$
$$4l + 40 = 65$$
$$4l = 25$$
$$l = \frac{25}{4} = 6.25$$
$$w = l+20 = 26.25$$

$$\boxed{\begin{aligned} \text{length} &= 6.25 \text{ ft} \\ \text{width} &= 26.25 \text{ ft} \end{aligned}}$$

Problem 7: (10pts) Complete the square for $f(x) = x^2 + 4x + 6$ and factor $f(x)$ completely.

$$\begin{aligned} f(x) &= x^2 + 4x + 6 \\ &= (x+2)^2 - 4 + 6 \\ &= \boxed{(x+2)^2 + 2} \end{aligned}$$

(this is as far as we can go w/o complex numbers, could also use $b^2 - 4ac = 16 - 4(6) = -8 < 0$ to see this)

Problem 8: (10pts) Solve $x^2 + 6x - 20 = 0$

completed square

$$(x+3)^2 - 9 - 20 = 0$$
$$(x+3)^2 - 29 = 0$$
$$(x+3-\sqrt{29})(x+3+\sqrt{29}) = 0$$
$$\boxed{x = -3 \pm \sqrt{29}}$$

Problem 9: (60pts) Factor each polynomial below completely over $\mathbb{R}$,

(a.) $x^3 - 16x$ $= x(x^2 - 16)$

$= x(x^2 - 4^2)$

$= \boxed{x(x-4)(x+4)}$

(b.) $x^2 + 6x + 9$ $= (x+3)(x+3) = \boxed{(x+3)^2}$

(c.) $2x^2 + 15x + 7$ $= \boxed{(2x+1)(x+7)}$

(d.) $x^3 - 1$ $= \boxed{(x-1)(x^2+x+1)}$

$A^3 - B^3 = (A-B)(A^2+AB+B^2)$ with $A = x$, $B = 1$.

(e.) $x^4 - 16$ $= (x^2)^2 - 4^2$

$= (x^2-4)(x^2+4)$

$= (x^2-2^2)(x^2+4)$

$= \boxed{(x-2)(x+2)(x^2+4)}$

(f.) $x^4 - x^2 - 6$ $= (x^2 - 3)(x^2 + 2)$

$= (x^2 - (\sqrt{3})^2)(x^2+2)$

$= \boxed{(x-\sqrt{3})(x+\sqrt{3})(x^2+2)}$