| MATH 101: FALL 2020 | TEST 3 |
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*You are allowed one page of notes and a calculator. No phones. More than 150pts to earn.* $\boxed{Box}$ *your answers for full credit and show work. Thanks!*

**Problem 1:** (10pts) Find domain in interval notation for $f(x) = \dfrac{2x+13}{x-6}$

$$X \neq 6 \quad \text{for} \quad f(x) = \frac{2X+13}{X-6} \in \mathbb{R}.$$

$$\boxed{\text{dom}\,(f(x)) = (-\infty, 6) \cup (6, \infty)}$$

**Problem 2:** (10pts) Simplify $\dfrac{16x^{99}}{2x^{11}} = 8X^{99-11} = \boxed{8X^{88}}$

**Problem 3:** (10pts) Simplify $\dfrac{\frac{1}{x}-7}{\frac{1}{x}+2} = \dfrac{X\left(\frac{1}{x}-7\right)}{X\left(\frac{1}{x}+2\right)} = \boxed{\dfrac{1-7X}{1+2X}}$

**Problem 4:** (10pts) Simplify $\dfrac{x^2}{5x+20} \cdot \dfrac{7x+28}{x^3}$ $= \dfrac{x^2 \, (7)(x+4)}{5(x+4)x^3}$

$$= \frac{7x^2}{5x^3}$$

$$= \boxed{\frac{7}{5x}}$$

**Problem 5:** (10pts) Simplify $\dfrac{x^2-16}{x^2+3x+2} \cdot \dfrac{x^2-4}{x-4}$

$$\left(\frac{x^2-16}{x^2+3x+2}\right) \cdot \left(\frac{x^2-4}{x-4}\right) = \frac{(x-4)(x+4)(x-2)(x+2)}{(x+1)(x+2)(x-4)}$$

$$= \boxed{\frac{(x+4)(x-2)}{x+1}} = \boxed{\frac{x^2+2x-8}{x+1}}$$

**Problem 6:** (10pts) Simplify $\dfrac{10x^2 + 20x^5 - 10x^6}{2x^2}$

$$\frac{10x^2 + 20x^5 - 10x^6}{2x^2} = \frac{10x^2}{2x^2} + \frac{20x^5}{2x^2} - \frac{10x^6}{2x^2}$$

$$= \boxed{5 + 10x^3 - 5x^4}$$

**Problem 7:** (10pts) Simplify $\dfrac{8a^2}{16a^5 - 24a^3 + 8a^2}$

$$\frac{8a^2}{16a^5 - 24a^3 + 8a^2} = \frac{8a^2}{8a^2(2a^3 - 3a + 1)} = \boxed{\frac{1}{2a^3 - 3a + 1}}$$

**Problem 8:** (10pts) Simplify $\sqrt[3]{-8x^{15}} = \sqrt[3]{-1}\,\sqrt[3]{8}\,\sqrt[3]{x^{15}}$

$$= (-1)(2)\, x^{15/3}$$

$$= \boxed{-2x^5}$$

**Problem 9:** (10pts) Rewrite the expression as to rationalize the denominator: $\dfrac{46}{7+\sqrt{3}}$

$$\frac{46}{7+\sqrt{3}}\left(\frac{7-\sqrt{3}}{7-\sqrt{3}}\right) = \frac{46\,(7-\sqrt{3})}{49-7\sqrt{3}+7\sqrt{3}-3} = \frac{46\,(7-\sqrt{3})}{46} = \boxed{7-\sqrt{3}}$$

**Problem 10:** (10pts) Find the Cartesian form of the complex number $\dfrac{20}{1+3i}$

$$\frac{20}{1+3i} = \left(\frac{20}{1+3i}\right)\left(\frac{1-3i}{1-3i}\right) = \frac{20\,(1-3i)}{1-3i+3i-(3i)^2} = \frac{20\,(1-3i)}{10}$$

$$= 2\,(1-3i)$$
$$= \boxed{2-6i}$$

$$(3i)^2 = 9i^2 = -9.$$

**Problem 11:** (15pts) Solve $\dfrac{x}{5} - \dfrac{x}{7} = 4$

$$\frac{7x-5x}{35} = 4$$
$$2x = 4\,(35)$$
$$x = 2\,(35)$$
$$\boxed{x = 70}$$

**Problem 12:** (15pts) Solve $\dfrac{x+3}{x+1} = \dfrac{5}{3}$.

$$3(x+3) = 5(x+1)$$
$$3x+9 = 5x+5$$
$$4 = 2x$$
$$\boxed{x = 2}$$

**Problem 13:** (15pts) Solve $\sqrt{x+7} - 2 = 5$.

$$\left(\sqrt{x+7}\right)^2 = (7)^2$$
$$x+7 = 49$$
$$\boxed{x = 42}$$

( check it, $\sqrt{42+7} - 2 = \sqrt{49} - 2 = 7 - 2 \overset{\checkmark}{=} 5$

**Problem 14:** (15pts) Solve $x = 7 + \sqrt{x-5}$.

$$x - 7 = \sqrt{x-5}$$
$$(x-7)^2 = \left(\sqrt{x-5}\right)^2$$
$$x^2 - 14x + 49 = x - 5$$
$$x^2 - 13x + 54 = 0$$
$$\left(x - \tfrac{13}{2}\right)^2 - \tfrac{169}{4} + \tfrac{54(4)}{4} = 0$$
$$\left(x - \tfrac{13}{2}\right)^2 = \tfrac{169 - 216}{4} < 0$$
$$\Rightarrow \boxed{\text{No REAL Solutions}}$$

oops!

well... the cost of one arithmetic mistake...

$$\rightarrow x^2 - 15x + 54 = 0$$
$$(x-9)(x-6) = 0$$
$$x = 6 \quad \text{or} \quad x = 9$$

However,
$$6 - 7 = -1 \neq \sqrt{6-5}$$

whereas,
$$9 - 7 = \sqrt{9-5}$$
$$2 \overset{\checkmark}{=} 2$$

$\therefore \boxed{x = 9}$