Examples for Law of Exponent Problem

E1 $\dfrac{\sqrt{x^2\sqrt{xy}}}{(x^3y)^2y^4} = x^A y^B$ Assume $x, y > 0$

$$\frac{\sqrt{x^2\sqrt{xy}}}{(x^3y)^2y^4} = \frac{\sqrt{x^2}\sqrt{\sqrt{xy}}}{(x^3)^2y^2y^4}$$

$$= \frac{(x^2)^{1/2}((xy)^{1/2})^{1/2}}{x^6y^2y^4}$$

$$= \frac{x(xy)^{1/4}}{x^6y^6}$$

$$= \frac{x\,x^{1/4}y^{1/4}}{x^6y^6}$$

$$= x^{1+\frac{1}{4}-6}\,y^{\frac{1}{4}-6}$$

$$= x^{-19/4}\,y^{-23/4}$$

Compare to $x^A y^B$

$$A = \frac{-19}{4} \quad \& \quad B = \frac{-23}{4}$$

$\frac{1}{4} - 5 = \frac{1-20}{4} = \frac{-19}{4}$

$\frac{1}{4} - 6 = \frac{1-24}{4} = \frac{-23}{4}$

E2 $\dfrac{x^{-3}y^2\sqrt[3]{xy}}{(xy)^{-2}x} = \dfrac{y^B}{x^A}$

$$\frac{x^{-3}y^2\sqrt[3]{xy}}{(xy)^{-2}x} = \frac{x^{-3}y^2(xy)^{1/3}}{x^{-2}y^{-2}x}$$

$$= \frac{x^{-3}y^2x^{1/3}y^{1/3}}{x^{-2}y^{-2}x^1}$$

$$= \frac{y^{2+\frac{1}{3}+2}}{x^{-2+1+3-1/3}}$$

$$= \frac{y^{4+1/3}}{x^{2-1/3}} = \frac{y^B}{x^A}$$

$$B = 4 + \frac{1}{3} = \frac{12+1}{3} = \frac{13}{3}$$

$$A = 2 - \frac{1}{3} = \frac{6-1}{3} = \frac{5}{3}$$

Reminder: Laws of Exponents

For $x, y > 0$,

$(xy)^t = x^t y^t$	$x^{-t} = \frac{1}{x^t}$	$x^{1/n} = \sqrt[n]{x}$
$(x^s)^t = x^{st}$	$\frac{1}{x^{-t}} = x^t$	$x^{1/2} = \sqrt{x}$