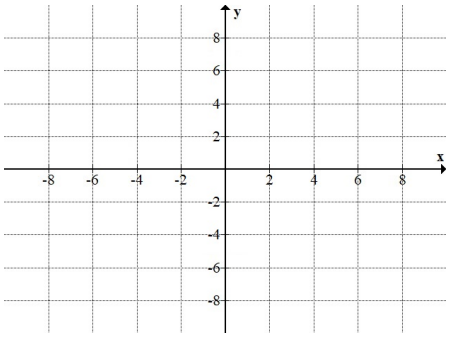
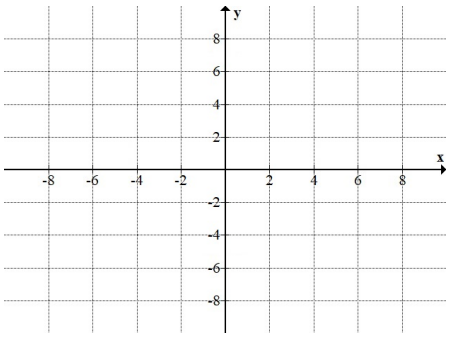
Please print this out and write your solutions on this document. I will only give half credit if the solutions are not written on this form. Please staple when finished. 60pts to earn here. Thanks!

Problem 46: (6pts) *The disciminant for* $f(x) = ax^2 + bx + c$ *is* $b^2 - 4ac$. *Recall, non-negative discriminant implies the quadratic polynomial can be factored over* $\mathbb{R}$ *whereas* $b^2 - 4ac < 0$ *implies* $ax^2 + bx + c$ *cannot be factored over* $\mathbb{R}$.
Calculate the discriminant for each $f(x)$ given below and factor $f(x)$ over $\mathbb{R}$ if possible. In addition, graph $y = f(x)$ carefully in the plot provided:

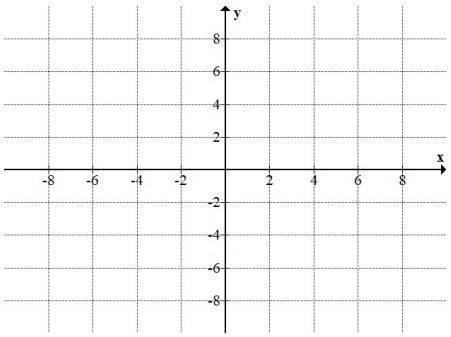
(a.) $f(x) = x^2 + 4x + 6$



(b.) $f(x) = x^2 + 8x + 12$

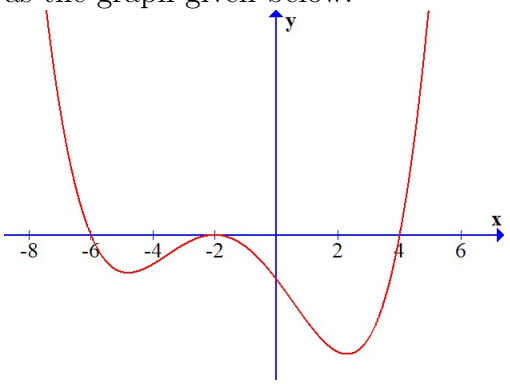


(c.) $f(x) = -x^2 + 8x - 16$

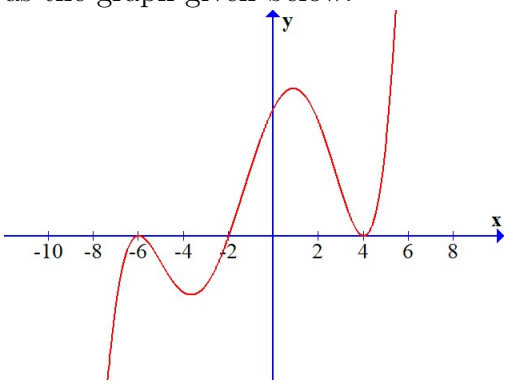


Problem 47: (2pts) Find a polynomial of least degree whose graph crosses the x-axis at $x = -4$ and $x = 3$ and bounces off the x-axis at $x = 1$. In addition, assume the y-intercept is 20. Find the formula for $P(x)$.

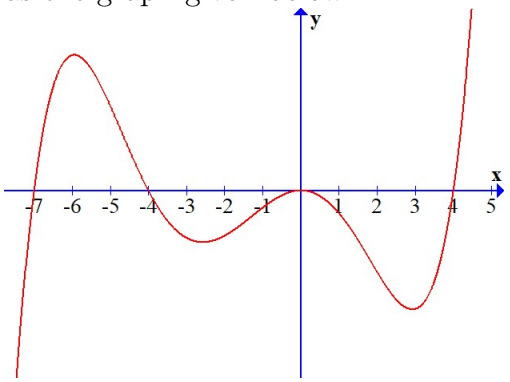
Problem 48: (2pts) Find $P(x)$ which could have a graph which shares the same shape and x-intercepts as the graph given below:



Problem 49: (1pts) Find $P(x)$ which could have a graph which shares the same shape and x-intercepts as the graph given below:



Problem 50: (1pts) Find $P(x)$ which could have a graph which shares the same shape and x-intercepts as the graph given below:



Problem 51: (2pts) Find a polynomial $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_o$ with zeros $-2, 0, 1, 3$ given that $a_3 = 4$.

Problem 52: (2pts) Let $P(x) = x^3 + 2x^2 - 9x - 18$. Show that -2 is a zero of $P(x)$ and find all the other zeros of $P(x)$. *Hint: factoring by grouping is a good idea here*

Problem 53: (2pts) Let $f(x) = x^4 + 2x^2 - 3x + 10$. Use long division to calculate $\dfrac{f(x)}{x^2+3}$.
Is $(x^2 + 3)$ a factor of $f(x)$?

Problem 54: (2pts) Let $f(x) = x^5 + 12x^2 - 3x + 2$. Calculate $\dfrac{f(x)}{x-1}$.
Is $(x - 1)$ a factor of $f(x)$?

Problem 55: (2pts) Factor $f(x) = x^5 - 3x^4 - 2x^3 + 6x^2 - 3x + 9$ completely over $\mathbb{R}$. Hint: $f(3) = 0$.

Problem 56: (2pts) Find the standard form of a polynomial with real coefficients of degree 4 which has complex zeros $3 + 2i$ and $7 - 3i$ with a y-intercept of 10.

Problem 57: (2pts) Factor $f(x) = x^4 + 7x^3 + 19x^2 + 23x + 10$ completely over $\mathbb{R}$. Hint: $f(-2+i) = 0$.

Problem 58: (2pts) State the rational roots theorem in your own words.

Problem 59: (2pts) If $R(x) = 2x^5 + 3x^3 + 4x^2 - 8$ then use the Rational Roots Theorem (aka the Rational Zeros Theorem) to list all possible rational zeros for $R(x)$.

Problem 60: (2pts) It is known that $P(x) = x^3 + 2x^3 - 13x + 10$ has real zeros which are integers. Factor $P(x)$ completely. *Hint: use the Rational Roots Theorem*

Problem 61: (2pts) Use Descartes' rule of signs to determine how many positive and how many negative real zeros there are for the polynomial $P(x) = 2x^6 + 5x^4 - x^3 - 5x - 1$.

Problem 62: (3pts) Factor the following polynomials completely over the complex numbers.

(a.) $x^2 - 4x + 5$

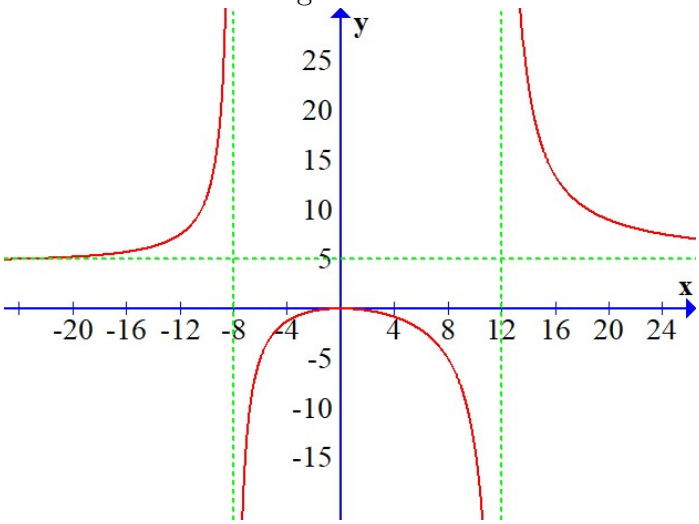
(b.) $x^4 + 4x^2 - 36$

(c.) $x^4 + x^2$

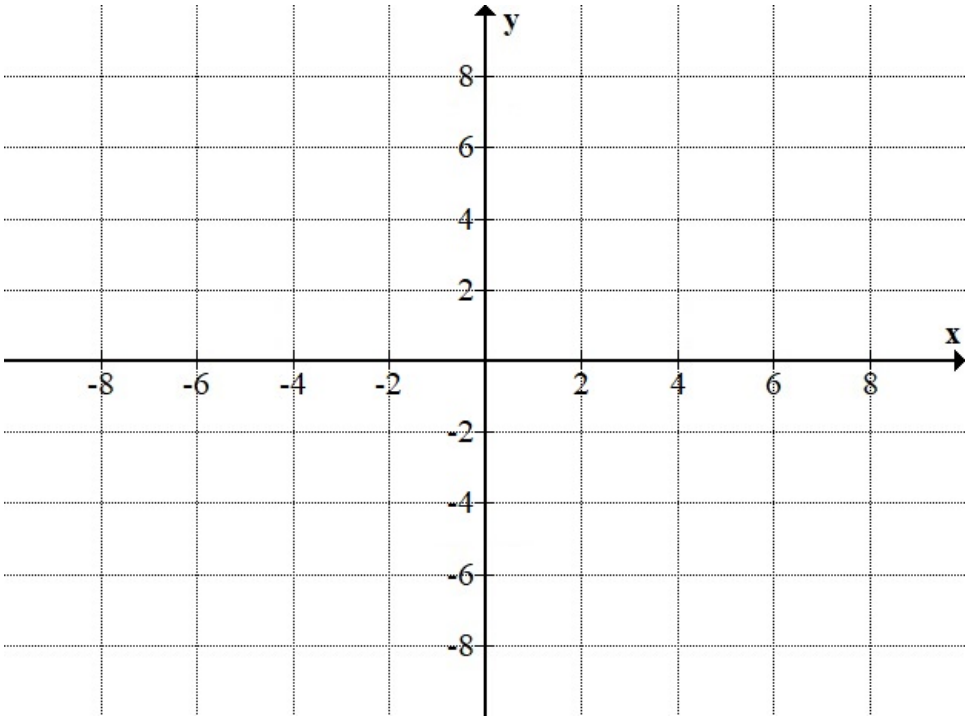
Problem 63: (2pts) Let $f(x) = (x^2 - 4)(x^2 - x - 2)^2$. Find all zeros of $f(x)$ and determine the multiplicity of each zero.

Problem 64: (2pts) Find the x and y-intercepts of $f(x) = \dfrac{x^2 - x - 2}{x - 6}$

Problem 65: (2pts) Write the equations for each horizontal and vertical asymptote for the graph of the rational function given below:



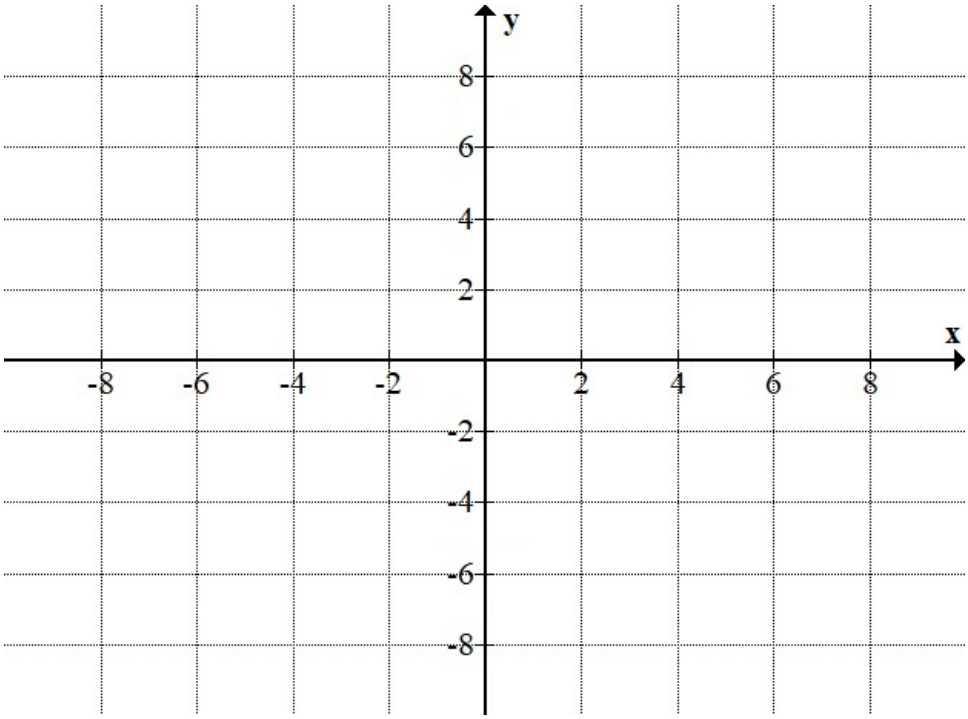
Problem 66: (4pts) Consider the rational function $f(x) = \dfrac{x^2 + 4x - 5}{x^2 + x - 2}$. Find all vertical or horizontal asymptotes, as well as any holes in the graph. Graph the function carefully with each feature clearly labeled.



Problem 67: (4pts) Consider the rational function

$$f(x) = 2 + \frac{(x-1)(x-6x+9)}{(x^2-2x+1)(x-3)(x^2-16)}.$$

Find all vertical or horizontal asymptotes, as well as any holes in the graph. Graph the function carefully with each feature clearly labeled.



Problem 68: (2pts) Solve $x^3 + 4x^2 \geq 4x + 16$. Write the answer in interval notation.

Problem 69: (3pts) Solve $\dfrac{x-3}{2x+5} \geq 1$. Write the answer in interval notation.

Problem 70: (2pts) Find all x for which the graph $f(x) = x^2$ lies above the graph of $g(x) = 3x + 10$.

Problem 71: (2pts) Find the domain of $h(x) = \sqrt[4]{x^4 - 1}$.