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MATH 113-08: FALL 2020 TEST 1

You are allowed one page of notes and a calculator. No phones. More than 25pts to earn. For full credit please BOX *your answers and show work. At least 150pts to earn here. Thanks!*

Problem 1: (10pts) Find the equation of a line whose graph contains points $\underbrace{(2,-1)}_{P_1}$ and $\underbrace{(0,5)}_{P_2}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-1)}{0-2} = \frac{6}{-2} = -3$$

Notice $(0,5)$ is y-intercept. Hence $\boxed{y = -3x + 5}$

Problem 2: (10pt) Multiply the following expressions and collect like power terms to give your answer as a polynomial in standard form:

$$\begin{aligned}(x+3)^2(x^2-1) &= (x+3)(x+3)(x^2-1) \\ &= (x^2+6x+9)(x^2-1) \\ &= (x^2+6x+9)x^2 - x^2 - 6x - 9 \\ &= x^4 + 6x^3 + 9x^2 - x^2 - 6x - 9 \\ &= \boxed{x^4 + 6x^3 + 8x^2 - 6x - 9}\end{aligned}$$

Problem 3: (10pt) Assume $x, y > 0$ and use laws of algebra to determine A, B as indicated below:

$$x^A y^B = \sqrt{\frac{x^{-4}(xy^5)^4}{(xy^3)^2}}$$

$$= \sqrt{\frac{x^{-4}x^4(y^5)^4}{x^2(y^3)^2}} \quad : \quad x^{-4}x^4 = \frac{x^4}{x^4} = 1.$$

$$= \sqrt{\frac{y^{20}}{x^2 y^6}}$$

$$= \sqrt{x^{-2}y^{14}}$$

$$= (x^{-2})^{1/2}(y^{14})^{1/2}$$

$$= \underline{x^{-1}y^7} \Rightarrow \boxed{A = -1} \text{ and } \boxed{B = 7}$$

Problem 4: (10pt) Solve $|2x+3|+2=13$.

$$|2x+3| = 11$$
$$2x+3 = \pm 11$$
$$2x = \pm 11 - 3$$
$$x = \frac{\pm 11 - 3}{2} = \frac{11-3}{2} \text{ or } \frac{-11-3}{2}$$
$$\therefore \boxed{x=4} \text{ or } \boxed{x=-7}$$

Problem 5: (20pt) **Factor** each $f(x)$ given below completely over $\mathbb{R}$:

(a.) $f(x) = x^3 - 9x^2 + 20x$

$$= x(x^2 - 9x + 20)$$
$$= \boxed{x(x-4)(x-5)}.$$

(b.) $f(x) = x^4 - 13x^2 + 36$

$$= (x^2-4)(x^2-9)$$
$$= \boxed{(x-2)(x+2)(x-3)(x+3).}$$

Problem 6: (10pt) Solve $\underbrace{|7-2x|<3}$ and write your answer in interval notation.

$$-3 < 7 - 2x < 3$$

subtract 7 across the compound inequality.

$$-10 < -2x < -4$$

divide by $-2 < 0$ flips the inequalities.

$$\frac{-10}{-2} > \frac{-2x}{-2} > \frac{-4}{-2}$$

$$5 > x > 2 \quad \text{aka} \quad 2 < x < 5$$

$$\Rightarrow x \in \boxed{(2,5)}$$

Problem 7: (10pts) Use completing the square and algebra as needed to place the circle equation below into standard form. Find the center and radius of the circle.

$$\underbrace{x^2-14x}+\underbrace{y^2+20y}+5=0$$

$$(x-7)^2-49+(y+10)^2-100+5=0$$

$$\boxed{(x-7)^2+(y+10)^2=144=12^2} \leftarrow \text{standard form}$$

Thus circle is $\boxed{\text{centered at } (7,-10) \text{ with radius } R=12.}$

Problem 8: (30pt) For each quadratic polynomial $f(x)$ given below, complete the square and find all real or complex solutions of $f(x)=0$:

(a.) $f(x) = 2x^2 + 8x + 10,$

$$= 2(x^2+4x+5)$$

$$= \boxed{2((x+2)^2+1)} \leftarrow \text{completed square.}$$

$$f(x) = 2[(x+2)^2+1] = 0$$

$$\Rightarrow (x+2)^2 = -1$$

$$\Rightarrow x+2 = \pm\sqrt{-1} = \pm i$$

$$\therefore \boxed{x = -2 \pm i}$$

(b.) $f(x) = x^2 - 6x - 4.$

$$= (x-3)^2 - 9 - 4$$

$$= \boxed{(x-3)^2 - 13} \leftarrow \text{completed square.}$$

$$f(x) = (x-3)^2 - 13 = 0$$

$$\Rightarrow (x-3)^2 = 13$$

$$\Rightarrow x-3 = \pm\sqrt{13}$$

$$\therefore \boxed{x = 3 \pm \sqrt{13}}$$

Problem 9: (10pt) Find real numbers a, b for which $a + ib = \dfrac{10}{1+3i}$.

$i^2 = -1$

$$a + ib = \frac{10(1-3i)}{(1+3i)(1-3i)} = \frac{10-30i}{1+3i-3i-9i^2} = \frac{10(1-3i)}{10} = \underline{1-3i}.$$

Thus $\underbrace{\boxed{a=1}}_{\text{real part of } \frac{10}{1+3i}}$ and $\underbrace{\boxed{b=-3}}_{\text{imaginary part of } \frac{10}{1+3i}}$

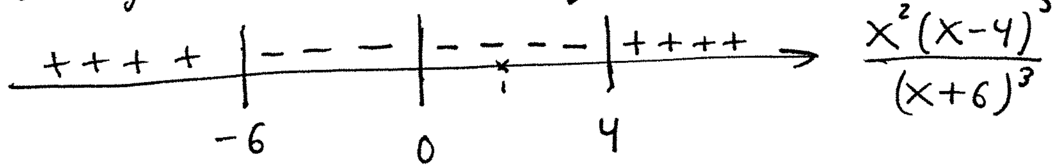
Problem 10: (20pts) Solve the following inequality using an appropriate technique. Show your work and write the answer using interval notation (you might need to use ∪ for union)

$$\frac{x^2(x-4)^5}{(x+6)^3} \geq 0$$

Since the rational expression above is already completely factored we can read off the algebraic critical #'s directly,

$-6, \ 0, \ 4$

Thus, as setting $x=1$ gives $\frac{1(-3)^5}{7^3} < 0$ thus



Remark: I put sign-flips at $x = -6$ and $x = 4$ because the corresponding factors $(x-4)^5$ and $(x+6)^3$ have odd powers. Alternatively, you can check a point in each subinterval.

In conclusion, from the above sign-chart and the fact $x = -6$ must be excluded due to division by zero,

$$\frac{x^2(x-4)^5}{(x+6)^3} \geq 0 \quad \text{for} \quad x \in \boxed{(-\infty, -6) \cup [4, \infty)}$$

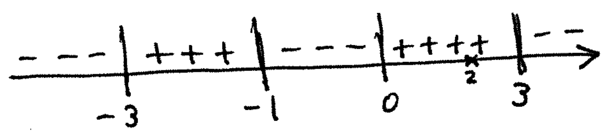
Problem 11: (10pts) Solve $\dfrac{1}{x+3} - \dfrac{2}{x-3} < 1$ and express your answer in interval notation using unions if appropriate.

$$\frac{1}{x+3} - \frac{2}{x-3} - 1 < 0$$

$$\frac{x-3}{(x+3)(x-3)} - \frac{2(x+3)}{(x+3)(x-3)} - \frac{\overbrace{(x+3)(x-3)}^{x^2-9}}{(x+3)(x-3)} < 0$$

$$\frac{x-3-2x-6-x^2+9}{(x+3)(x-3)} < 0$$

$$\frac{-x^2-x}{(x+3)(x-3)} = \frac{-x(x+1)}{(x+3)(x-3)}$$ evaluate at $x=2$ to obtain $\dfrac{-2(3)}{5(-1)} > 0$



$$\frac{-x(x+1)}{(x+3)(x-3)}$$

There fore the sol$^{\underline{n}}$ to $*$ is all points in $\boxed{(-\infty,-3)\cup(-1,0)\cup(3,\infty}$

Problem 12: (10pt Bonus) Let $P=(2,0)$ and $Q=(8,6)$ and $R=(3,3)$ be vertices of a triangle. Find the area and perimeter of this triangle.

I'll take sol$^{\underline{n}}$ for this problem up to the time of TEST 2.

THANKS!