

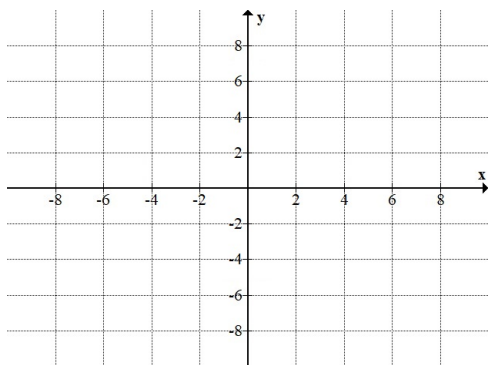
NAME: \_\_\_\_\_.

MATH 113:

TEST 3

*No phones. You are allowed a calculator and a sheet of notes front and back. At least 150pts to earn here. Thanks!*

**Problem 1:** (10pts) Factor  $f(x) = x^2 - 12x + 38$  over  $\mathbb{R}$  if possible, find the vertex of the parabola  $y = f(x)$ , and graph  $y = f(x)$  carefully in the plot provided:

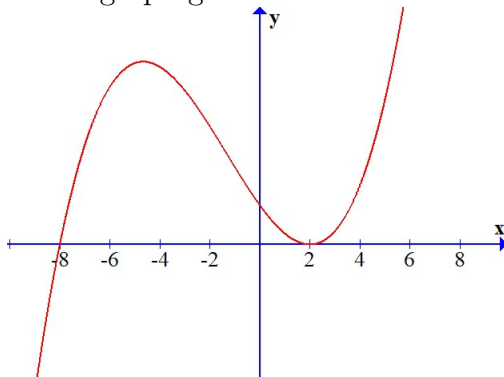


**Problem 2:** (15pts) Solve  $\frac{1}{x^2 + 2x - 3} > 0$  and write your answer using interval notation.

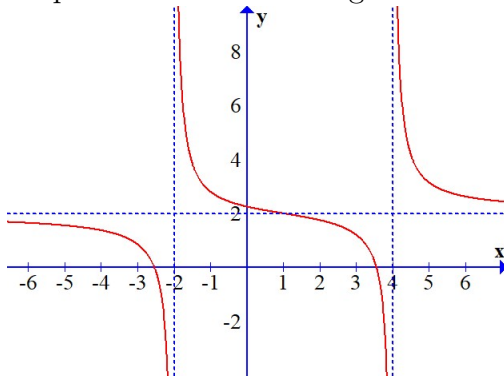
**Problem 3:** (15pts) Solve  $\frac{x+4}{x-3} \leq 1$ . Write the answer in interval notation.

**Problem 4:** (15pts) Suppose a polynomial  $P(x)$  has a graph which crosses the  $x$ -axis at  $x = -4$  and bounces off the  $x$ -axis at  $x = 5$ . Find formula of  $P(x)$  given that the  $y$ -intercept is  $-200$ .

**Problem 5:** (15pts) Find  $P(x)$  which could have a graph which shares the same shape and  $x$ -intercepts as the graph given below:



**Problem 6:** (10pts) Find a rational function  $f(x)$  which could have a graph which shares the same shape as well as matching horizontal and vertical asymptotes of the graph given below:



**Problem 7:** (15pts) Let  $P(x) = x^4 + 2x^3 - 5x^2 - 6x$ . Show that 2 is a zero of  $P(x)$  and factor  $P(x)$  completely over  $\mathbb{R}$ .

**Problem 8:** (15pts) Factor  $f(x) = x^4 - 2x^3 + x^2 + 18x - 90$  completely over  $\mathbb{R}$ . Hint:  $f(1 + 3i) = 0$ .

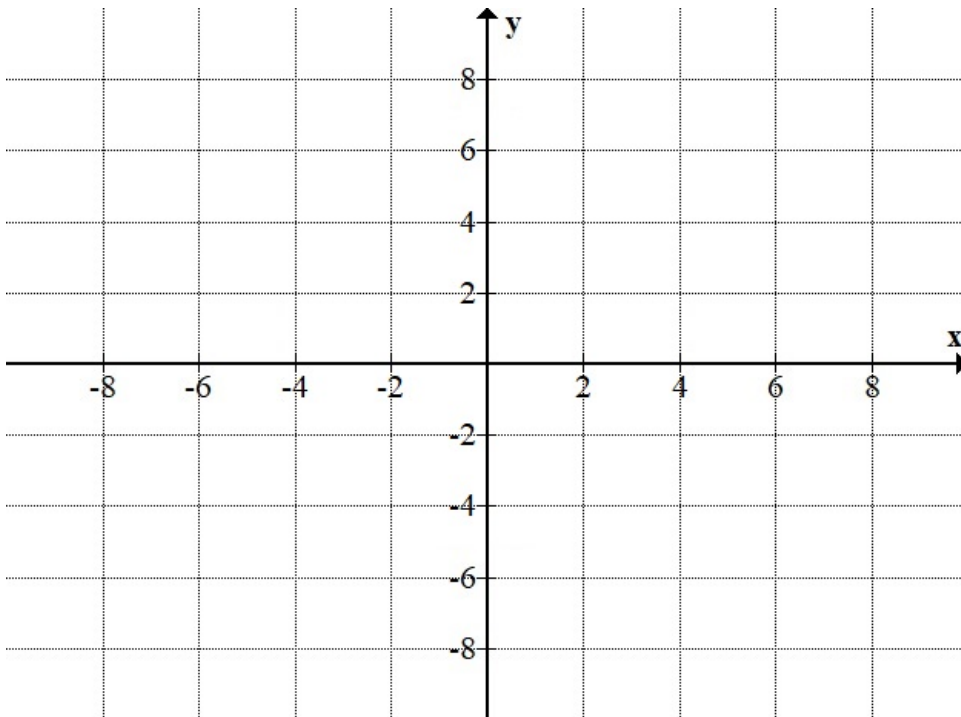
**Problem 9:** (15pts) It is known that  $P(x) = x^3 - 7x^2 + 7x + 15$  has real zeros which are integers. Factor  $P(x)$  completely. *Hint: use the Rational Roots Theorem*

**Problem 10:** (20pts) Factor the following polynomials completely over the complex numbers.

(a.)  $x^4 - 6x^3 + 11x^2$

(b.)  $x^4 - 8x^2 - 7$

**Problem 11:** (10pts) Consider the rational function  $f(x) = \frac{2x^2}{16x - x^3}$ . Find all vertical or horizontal asymptotes, as well as any holes in the graph. Graph the function carefully with each feature clearly labeled.



**Problem 12:** (5pts) Write the range of function in the previous problem in interval notation.