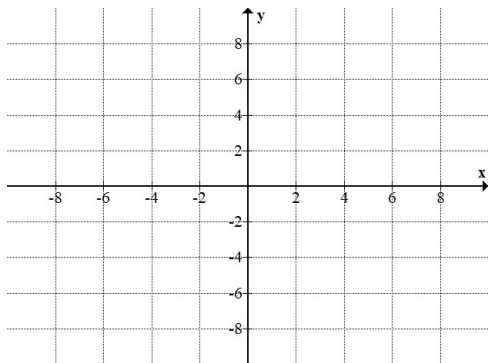


No phones. You are allowed a calculator and a sheet of notes front and back. At least 150pts to earn here. Thanks!

**Problem 1:** (10pts) Factor  $f(x) = x^2 + 12x + 30$  over  $\mathbb{R}$  if possible, find the vertex of the parabola  $y = f(x)$ , and graph  $y = f(x)$  carefully in the plot provided:

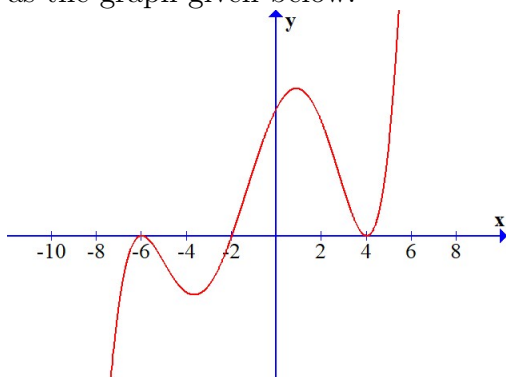


**Problem 2:** (15pts) Solve  $\frac{1}{x^2 - 9} > 0$  and write your answer using interval notation.

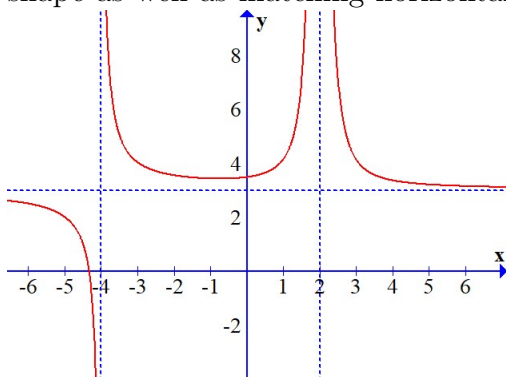
**Problem 3:** (15pts) Solve  $\frac{x}{x + 9} \leq 1$ . Write the answer in interval notation.

**Problem 4:** (15pts) Suppose a polynomial  $P(x)$  has a graph which crosses the  $x$ -axis at  $x = -7$  and bounces off the  $x$ -axis at  $x = 3$ . Find formula of  $P(x)$  given that the  $y$ -intercept is 42.

**Problem 5:** (15pts) Find  $P(x)$  which could have a graph which shares the same shape and  $x$ -intercepts as the graph given below:



**Problem 6:** (10pts) Find a rational function  $f(x)$  which could have a graph which shares the same shape as well as matching horizontal and vertical asymptotes of the graph given below:



**Problem 7:** (15pts) Let  $P(x) = x^5 + 2x^4 - 81x - 162$ . Show that  $-2$  is a zero of  $P(x)$  and factor  $P(x)$  completely over  $\mathbb{R}$ .

**Problem 8:** (15pts) Factor  $f(x) = x^5 - 9x^4 + 37x^3 - 67x^2 + 54x - 16$  completely over  $\mathbb{R}$ . Hint:  
 $f(3 + i\sqrt{7}) = 0$ .

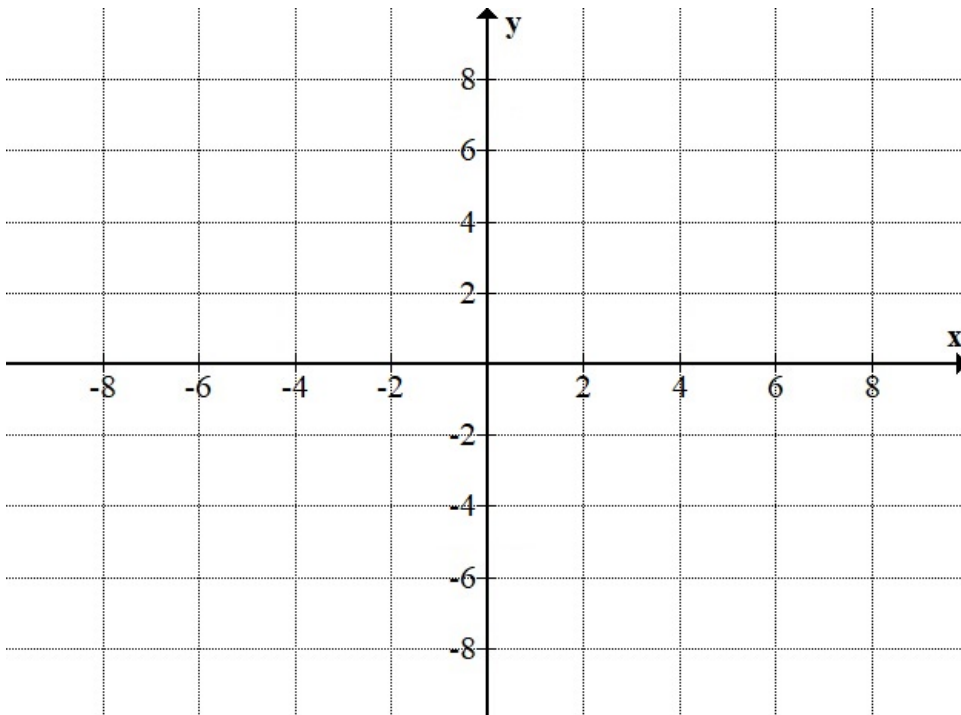
**Problem 9:** (15pts) It is known that  $P(x) = x^4 - 16x^3 + 86x^2 - 176x + 105$  has real zeros which are integers. Factor  $P(x)$  completely. *Hint: use the Rational Roots Theorem;  $105 = 3 \cdot 5 \cdot 7$*

**Problem 10:** (20pts) Factor the following polynomials completely over the complex numbers.

(a.)  $x^4 - 7x^3 + 9x^2$

(b.)  $x^4 - 7x^2 - 8$

**Problem 11:** (10pts) Consider the rational function  $f(x) = \frac{2x^3}{16x - x^3}$ . Find all vertical or horizontal asymptotes, as well as any holes in the graph. Graph the function carefully with each feature clearly labeled.



**Problem 12:** (5pts) Write the range of function in the previous problem in interval notation.