NAME ____

MATH 114: FALL 2021 — FINAL EXAM

You may use the provided unit-circle and formula sheet. You are also allowed a page of notes.

Problem 1: (5pts) Suppose $\theta = 2\pi/9$ (in radians). Convert this angle to degrees.

Problem 2: (5pts) Suppose $\theta = 150^o$. Convert this angle to radians.

Problem 3: (5pts) Find the angle $0 < \alpha < 360^o$ which is coterminal with $\theta = -40^o$.

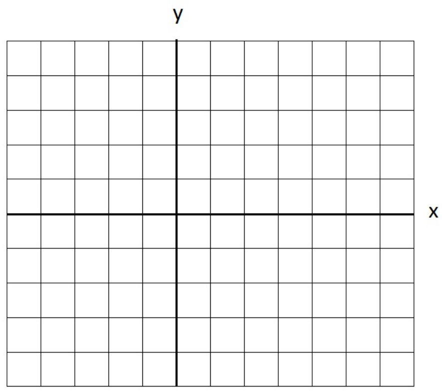
Problem 4: (10pts) Consider a radius $10\,cm$ circular arc which sweeps through a 150^o. Find:

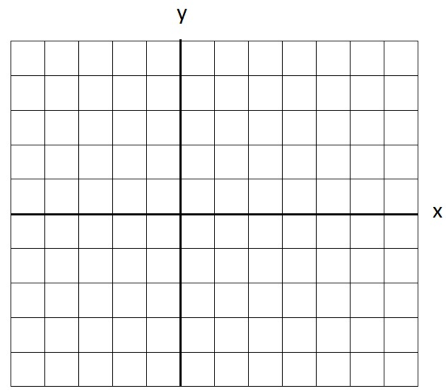
(a.) the arclength of the arc,

(b.) the area of the sector,

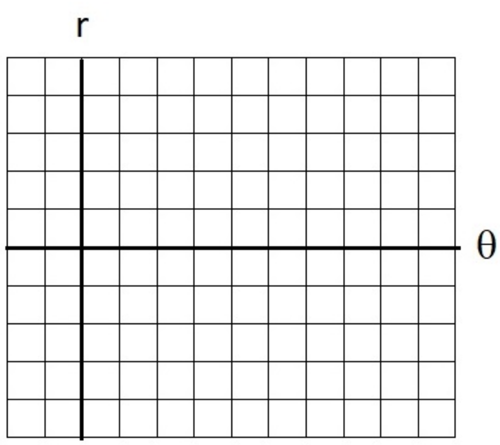
Problem 5: (10pts) If $\cos t = -2/3$ and t is an angle in quadrant III then find the exact value of $\sin t$.

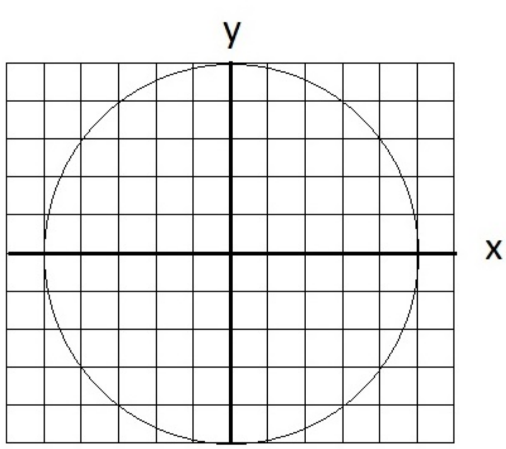
Problem 6: (10pts) Graph $y = 3\sin\left(\frac{\pi x}{2}\right)$ in the left grid and $y = 2\cos\left(\frac{\pi x}{2}\right) - 2$ in the right grid provided below. Also, answer the following question: what is the period of these sinusoidal functions ?





Problem 7: (10pts) Graph $r = 5\sin(3\theta)$ using the grids given below:





Problem 8: (10pts) If α, β and γ are angles in the same triangle, then prove that $\sin(\alpha+\beta) = \sin\gamma$.

Problem 9: (5pts) Simplify $\sin(-x)\sec(-x)\csc(-x)$.

Problem 10: (10pts) Simplify the expression below and leave your answer in terms of $\sin x$.

$$\frac{\sec x + \csc x}{1 + \tan x}$$

Problem 11: (10pts) Use trigonometric identities to simplify the following expression:

$$\frac{\sin x \cos^2 x + \sin^3 x}{\csc x} + \cos^2 x$$

Problem 12: (15pts) Use an appropriate identity to rewrite each of the following expressions:

(A.) $\cos 10x \cos 8x + \sin 10x \sin 8x =$

(B.) $\cos 4x \sin 7x + \sin 4x \cos 7x =$

(C.) $\sin 2x + \sin 5x =$

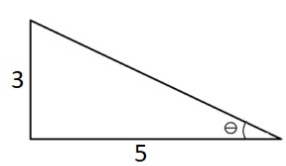
Problem 13: (10pts) Find the exact value of $\tan\left(\sin^{-1}\left(\frac{1}{x}\right)\right)$ in terms of x.

Problem 14: (10pts) Write the range of each inverse function in interval notation on the blanks provided:

(A.) $range(\tan^{-1}) =$ ______________

(B.) $range(\cos^{-1}) =$ ______________

Problem 15: (10pts) Find the length of the hypotenuse and the angle θ in the triangle pictured below:



Problem 16: (15pts) Find a solution in degrees or state no solution exists.

(A.) $\cos x = 1.001$

(B.) $\sin x = 0.4$

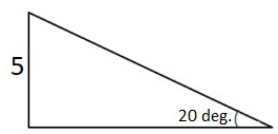
(C.) $\tan x = 1$

Problem 17: (10pts) Solve $\cos(2x) = \frac{1}{2}$ for $x \in [0, 2\pi]$.

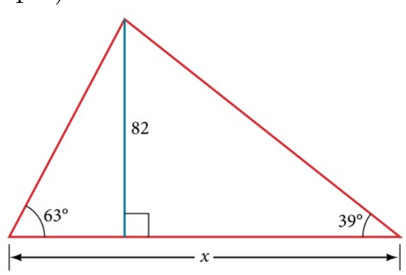
Problem 18: (10pts) Find all solutions of $2\sin^2 x - 5\sin x + 2 = 0$ for $x \in [0, \pi]$.

Problem 19: (5pts) Find the polar form of the equation $x^2 + 3y + y^2 = 0$.

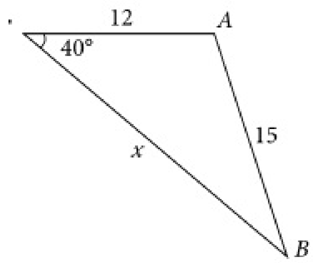
Problem 20: (10pts) Find the length of the hypotenuse and the adjacent side of the triangle below:



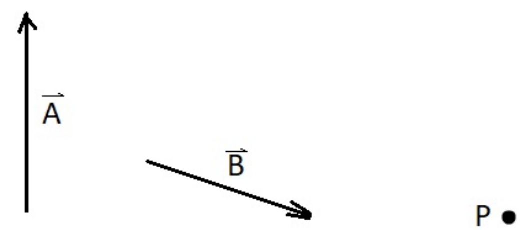
Problem 21: (10pts) Find x.



Problem 22: (10pts) Find x.



Problem 23: (10pts) Vectors $\vec{A}$ and $\vec{B}$ are plotted below. Draw $\vec{A}+\vec{B}$ starting at the point P pictured below. Use the tip-to-tail method.



Problem 24: (20pts) Find the standard angle (in degrees) and magnitude of each of the following vectors:

(a.) $\vec{C} = \left\langle -2, 2\sqrt{3} \right\rangle$

(b.) $\vec{D} = \langle 1, -1 \rangle$

Problem 25: (10pts) If $\vec{A}$ has $A = 10$ and standard angle 30^o and $\vec{B}$ has $B = 5$ and standard angle 270^o then find the magnitude and standard angle of $\vec{A} + \vec{B}$.

Problem 26: (10pts) Write the following complex numbers in polar form ($z = re^{i\theta}$).

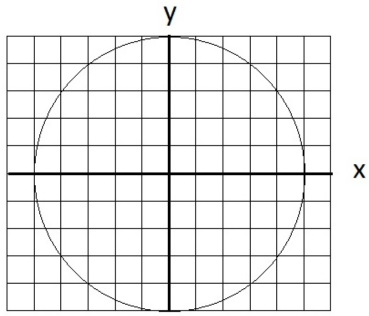
(a.) $z = 1 + i$,

(b.) $z = -3i$,

Problem 27: (10pts) Let $z = 2 + 3i$ and $w = -1 - 2i$. Find the Cartesian and polar forms of $(z + w)^8$.

Problem 28: (5pts) Use the formulas $\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$ and $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$ to derive the identity
$\cos(2x)\sin(3x) = \dfrac{1}{2}\sin(x) + \dfrac{1}{2}\sin(5x)$.

Problem 29: (10pt) Let $z = 125e^{3\pi i/6}$. Find the Cartesian form of $\sqrt[3]{z}$ and plot all three elements of $z^{1/3}$ in the plot below:



Problem 30: (20pts) Let $P = (-4, -4)$ and $Q = (2, 4)$ and $R = (6, -2)$. Find the perimeter, interior angles, and area of the triangle PQR. Is this triangle oblique ?