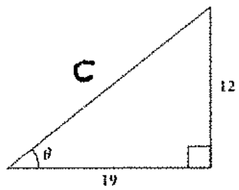
NAME ______________________________

MATH 114: FALL 2021	QUIZ 3 (10PTS+10PTS BONUS)

You may use your homework solutions. I need to look at your class notes while you take this. You are allowed a 3x5 inch card of formulas. Thanks! 2pts per problem.

Problem 1: Find the length of the hypotenuse and the angle θ in the triangle pictured below:



$C = \sqrt{12^2 + 19^2} = \boxed{\sqrt{505}}$ ← hypotenuse length

$\theta = \tan^{-1}\left(\frac{12}{19}\right) \cong \boxed{0.56 \text{ rad} = 32.3^\circ}$

Problem 2: Given $\sin\theta = -1/2$ and $\cos\theta = \sqrt{3}/2$ find $\sec\theta$.

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{3}/2} = \boxed{\frac{2}{\sqrt{3}}}$

Problem 3: Simplify $\cos 41x \cos x - \sin 41x \sin x$.

$\cos 41x \cos x - \sin 41x \sin x = \cos(41x + x) = \boxed{\cos(42x)}$

Problem 4: Simplify $\sin 4x \cos 3x - \cos 4x \sin 3x$.

$\sin 4x \cos 3x - \cos 4x \sin 3x = \sin(4x - 3x) = \boxed{\sin(x)}$

Problem 5: Simplify $\sin 7x + \sin 5x$ using one of the sum to product identities.

$$\sin 7x + \sin 5x = 2 \sin\left(\frac{7x+5x}{2}\right) \cos\left(\frac{7x-5x}{2}\right)$$
$$= \boxed{2\sin(6x)\cos(x)}$$

Problem 6: Simplify $\sin 4x \cos 11x$ using one of the product to sum identities.

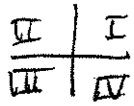
$$\sin 4x \cos 11x = \frac{1}{2}\left(\sin(4x+11x) + \sin(4x-11x)\right)$$
$$= \frac{1}{2}\left(\sin(15x) + \sin(-7x)\right) = \boxed{\frac{1}{2}[\sin(15x) - \sin(7x)]}$$

Problem 7: Use trigonometric identities to simplify the expression below:

$-2\cos(-x)\sin(-x) = -2\cos(x)(-\sin x)$: cosine even / sine odd

$= 2\cos x \sin x$

$= \boxed{\sin(2x)}$



$\pi < \theta < \frac{3\pi}{2}$ or $-\pi < \theta < -\frac{\pi}{2}$

$2\pi < 2\theta < 3\pi$ or $-2\pi < 2\theta < -\pi$

Problem 8: Find exact value of $\sin(2\theta)$ given that $\sin\theta = -1/5$ and θ is in Quadrant III.

$$\sin^2\theta + \cos^2\theta = 1 \Longrightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta} = -\sqrt{1-\tfrac{1}{25}} = -\frac{\sqrt{24}}{5}.$$

$$\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(\frac{-1}{5}\right)\left(-\frac{\sqrt{24}}{5}\right) = \boxed{+\frac{2\sqrt{24}}{25} = +\frac{4\sqrt{6}}{25}}$$

Problem 9: Use trigonometric identities to rewrite the expression below in terms of a sum of cosine functions with various arguments:

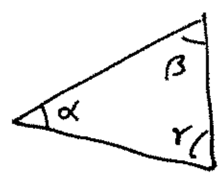
$$\sin^4(\theta) = \tfrac{1}{2}(1-\cos(2\theta))\,\tfrac{1}{2}(1-\cos(2\theta)) \quad : \text{ since } \sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta) \text{ and } \sin^4\theta = \sin^2\theta\sin^2\theta$$

$$= \tfrac{1}{4}\left[1 - 2\cos(2\theta) + \cos^2(2\theta)\right]$$

$$= \tfrac{1}{4}\left[1 - 2\cos(2\theta) + \tfrac{1}{2}(1+\cos(4\theta))\right]$$

$$= \tfrac{1}{4}\left[\tfrac{3}{2} - 2\cos(2\theta) + \tfrac{1}{2}\cos(4\theta)\right]$$

$$= \boxed{\tfrac{3}{8} - \tfrac{1}{2}\cos(2\theta) + \tfrac{1}{8}\cos(4\theta)}$$



$\alpha + \beta + \gamma = \pi$

Problem 10: If α, β and γ are angles in the same triangle, then prove that $\sin(\alpha+\beta) = \sin\gamma$.

$$\sin(\alpha+\beta) = \sin(\pi - \gamma) \quad : \text{since } \alpha+\beta+\gamma = \pi \Rightarrow \underline{\alpha+\beta = \pi - \gamma}.$$

$$= \cancel{\underbrace{\sin\pi}_{0}}\cos(-\gamma) + \underbrace{\cos\pi}_{-1}\sin(-\gamma) \quad : \text{adding angles formula for sine}$$

$$= -(-\sin(\gamma)) \quad : \text{since sine is odd.}$$

$$= \underline{\sin(\gamma)} \;//$$