The text for this course is *Mathematical Analysis I* second edition by Beatriz Laferriere, Gerardo Laferriere and Nguyen Mau Nam.

Problem 1: Tell me something you learned from reading the article by Pete Clark.

Problem 2: Let A, B, C, D be sets. Prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Problem 3: Let A, B, C, D be sets where $A \subseteq C$ and $B \subseteq D$. Prove $A \times B \subseteq C \times D$.

Problem 4: Let A, B, C be sets. Prove $A - (B \cup C) = (A - B) \cap (A - C)$.

Problem 5: Let $B, C \subseteq X$ where X is the universal set. Prove $\overline{B \cup C} = \overline{B} \cap \overline{C}$.
Hint: you may reference the result of the previous problem

Problem 6: Consider A, B, C finite sets. Let $\text{card}(A) = |A|$ denote the number of elements in A. Consider using an appropriate picture (Venn Diagram) to solve the following:

(a.) Explain why $|A \cup B| = |A| + |B| - |A \cap B|$.

(b.) Explain why $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

There are similar formulas for unions of more sets, however, Venn Diagrams are only easy to draw for up to 3 sets.

Problem 7: Let $F : \mathbb{R}^{2\times 2} \to \mathbb{R}$ be defined by $F\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$. Prove or disprove that F is one-to-one. Prove or disprove that F is onto.

Problem 8: Exercise 1.2.2 from the text.

Problem 9: Exercise 1.2.3 from the text.

Problem 10: Exercise 1.2.4 from the text.

Problem 11: Exercise 1.2.7 (just part (d)) from the text.

Problem 12: Exercise 1.2.8 (just part (c)) from the text.

Problem 13: Let $A = [0, 2]$ and $B = \{1, 2, 3, 4\}$ define a relation on $\mathbb{R}$ by $R = A \times B \subseteq \mathbb{R} \times \mathbb{R}$.

(a.) find the domain of R

(b.) find the range of R

(c.) is R a function ?

Problem 14: Define $C_k \subseteq \mathbb{R}^2$ by $C_k = F^{-1}(\{k\})$ where $F(x, y) = x^2 + y^2$ and $k \in [0, \infty)$. For each $(x_1, y_1), (x_2, y_2) \in \mathbb{R}^2$, define $(x_1, y_1)R(x_2, y_2)$ if and only if there exists $k \in [0, \infty)$ such that $(x_1, y_1), (x_2, y_2) \in C_k$. Prove R is an equivalence relation on $\mathbb{R}^2$. Also, describe the equivalence classes of R and how they form a partition of $\mathbb{R}^2$.

Problem 15: Let $x, y \in \mathbb{Z}$ be R-related iff $y - x \in 3\mathbb{Z} = \{3k \mid k \in \mathbb{Z}\}$. Prove R is an equivalence relation on $\mathbb{Z}$. Also, describe the equivalence classes of R and how they partition $\mathbb{Z}$.

Problem 16: Suppose A and B each have n-elements. Prove a function $f : A \to B$ is injective iff f is surjective.

Problem 17: Suppose A and B are infinite sets with the same cardinality and suppose $f : A \to B$ is a function. If f is injective then is f surjective ? Likewise, if f is surjective then is f injective ? Discuss.

Problem 18: Explain how the cardinalities of the sets below are related. In particular, place the sets in order from smallest to greatest cardinality.

$$\mathbb{R}, (0, \infty), \mathbb{N}, [3, 7], \mathbb{Q}, \mathcal{P}(\mathbb{R}), \mathbb{Q} \times \mathbb{Q}, \{1, 2, 3, 4\}, \mathcal{P}(\{a, b\}), \emptyset$$

Problem 19: Find a bijection from $[0, 1]$ to $[4, 8]$.

Problem 20: Find a bijection from $(-\pi/2, \pi/2)^2$ to $\mathbb{R}^2$.