The text for this course is *Mathematical Analysis I* second edition by Beatriz Laferriere, Gerardo Laferriere and Nguyen Mau Nam. The exercises below are from this text. This homework covers the material discussed in Lectures 19, 20, 21 and 22. It is due 11-9-20.

Problem 81: Exercise 4.1.1

Problem 82: Exercise 4.1.3

Problem 83: Exercise 4.1.4

Problem 84: Prove $\frac{d}{dx}\cos x = -\sin x$ given that $\lim_{h\to 0}\frac{\sin(h)}{h} = 1$ and $\lim_{h\to 0}\frac{1-\cos(h)}{h} = 0$.

Problem 85: Prove $\frac{d}{dx}\sin x = \cos x$, once more assume the limits given in the problem above are known.

Problem 86: Let $f(x) = x\sqrt{x^2}$ for $x \in \mathbb{R}$. Prove f is differentiable on $\mathbb{R}$. Explain why f is not twice differentiable on $\mathbb{R}$.

Problem 87: Exercise 4.1.11

Problem 88: Exercise 4.1.12

Problem 89: Exercise 4.1.13

Problem 90: Exercise 4.2.1

Problem 91: Exercise 4.2.2 part b (note to self: prove $\frac{d}{dx}e^x = e^x$ in Lecture 19)

Problem 92: Exercise 4.2.3

Problem 93: Exercise 4.2.4

Problem 94: Exercise 4.3.3

Problem 95: Exercise 4.3.5

Problem 96: Exercise 4.5.2

Problem 97: Exercise 4.5.3

Problem 98: A **series** $a_1+a_2+\cdots$ in $\mathbb{R}$ is said to converge to a real number S if the sequence of partial sums converges to S. That is, $a_1+a_2+\cdots = S$ if and only if $a_1+a_2+\cdots+a_n \to S$ as $n\to\infty$. A series which does not converge to a real number is said to be divergent. **Prove:** for $a, r \in \mathbb{R}$ the series $a+ar+ar^2+\cdots = \frac{a}{1-r}$ whenever $|r|<1$ and the series diverges for $|r| \geq 1$.

Problem 99: A sequence of functions $\{f_n\}_{n=1}^{\infty}$ where $f_n : I \subseteq \mathbb{R} \to \mathbb{R}$ is said to converge to $f : I \subseteq \mathbb{R} \to \mathbb{R}$ if for each $x \in I$ we have $f_n(x) \to f(x)$ as $n \to \infty$. Prove $f_n(x) = 1 + x + \cdots + x^{n-1}$ converges pointwise to $f(x) = \dfrac{1}{1-x}$ for $x \in (-1,1)$.

Problem 100: Let $x \in \mathbb{R}$. Prove $\displaystyle\sum_{k=0}^{n} \frac{(-1)^k}{(2k+1)!}x^{2k+1} \to \sin(x)$ as $n \to \infty$. (hint: use Taylor's Theorem)