Mission 6 Solution

$\boxed{P101}$ Let $f(x) = x^2 - a$ where $a > 0$. Newton's Method is given by $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. Calculate,

$$x_{n+1} = x_n - \frac{x_n^2 - a}{2x_n} = x_n - \frac{1}{2}x_n + \frac{a}{2x_n}$$

thus $\boxed{x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)}$. Suppose $x_0 > \sqrt{a} \Rightarrow x_0^2 > a$ and consider $\varphi(x) = \frac{1}{2}\left(x + \frac{a}{x}\right)$ notice φ has

$$\varphi'(x) = \frac{1}{2}\left(1 - \frac{a}{x^2}\right) \quad \& \quad \varphi''(x) = \frac{a}{x^3}$$

Notice $x \in [\sqrt{a}, x_0]$ gives $\varphi''(x) = \frac{a}{x^3} > 0$ thus φ' is an increasing function and $\varphi'(\sqrt{a}) \leq \varphi'(x) \leq \varphi'(x_0)$ for each $x \in (\sqrt{a}, x_0)$. Furthermore, $\varphi'(\sqrt{a}) = \frac{1}{2}(1 - \frac{a}{a}) = 0$! and $\varphi'(x_0) = \frac{1}{2}\left(1 - \frac{a}{x_0^2}\right) \therefore 0 \leq \varphi'(x) \leq \frac{1}{2}\left(1 - \frac{a}{x_0^2}\right)$.

If $x, y \in [\sqrt{a}, x_0]$ and $x < y$ then by MVT, $\exists c \in (\sqrt{a}, x_0)$ for which $\frac{\varphi(x) - \varphi(y)}{x - y} = \varphi'(c)$ thus $\left|\frac{\varphi(x) - \varphi(y)}{x - y}\right| \leq |\varphi'(x_0)|$

that is, $|\varphi(x) - \varphi(y)| \leq \frac{1}{2}\left(1 - \frac{a}{x_0^2}\right)|x - y|$ this suggests φ is contraction map with constant $k = \frac{1}{2}\left(1 - \frac{a}{x_0^2}\right)$. It remains to show $\varphi([\sqrt{a}, x_0]) \subseteq [\sqrt{a}, x_0]$. However, that is simple to show since φ is increasing on $[\sqrt{a}, x_0]$,

$$\varphi(\sqrt{a}) \leq \varphi(x) \leq \varphi(x_0)$$

$$\frac{1}{2}\left(\sqrt{a} + \frac{a}{\sqrt{a}}\right) = \sqrt{a} \leq \varphi(x) \leq \frac{1}{2}\left(x_0 + \frac{a}{x_0}\right) < \frac{1}{2}\left(x_0 + \frac{x_0^2}{x_0}\right) = x_0$$

Thus φ is contraction map on $[\sqrt{a}, x_0]$ with contraction constant $k = \frac{1}{2}\left(1 - \frac{a}{x_0^2}\right)$.


O'Charley's
HONEY

$\boxed{P102}$ to calculate $\sqrt{2}$ to 3 decimal places we need $|X_n - \sqrt{2}| < 0.0001$ to be nice and safe.

I'll guess

$X_0 = 1.5$ since $1.5^2 = 2.25$ is close to 2.

Contraction Map Th^{m} gives $|X_n - X_*| \leq \dfrac{k^n |X_0 - X_1|}{1-k}$

Where $X_{n+1} = \varphi(X_n)$ & $\varphi(x) = \frac{1}{2}(x + a/x)$

where $a = 2$ and $X_0 = 1.5$, Also,

$$k = \frac{1}{2}\left(1 - \frac{a}{X_0^2}\right) = \frac{1}{2}\left(1 - \frac{2}{2.25}\right) = \frac{1}{18}$$

Find $X_1 = \varphi(X_0) = \frac{1}{2}\left(1.5 + \frac{2}{1.5}\right) = \frac{17}{12} \cong 1.41666...$

Solve $\dfrac{k^n |X_0 - X_1|}{1-k} = \dfrac{1}{(18)^n} \dfrac{|1.5 - 17/12|}{1 - 1/18} = \dfrac{1}{(18)^n} \dfrac{\frac{1}{12}}{\frac{17}{18}} = \dfrac{1}{204\,(18)^{n-1}}$

Need $\dfrac{1}{(204)(18)^{n-1}} < 0.001$

$n=1$ gives $\frac{1}{204} \approx 0.0049$ as bound on error.

$n=2$ gives $\frac{1}{(204)18} \approx 0.00027$ as bound on error

$n=3$ gives $\frac{1}{(204)(18)^2} \approx 0.000015$ thus X_3 certainly suffices.

Hence calculate,

$$X_2 = \varphi(X_1) = \frac{1}{2}\left(\frac{17}{12} + \frac{2}{17/12}\right) = \frac{577}{408} \approx 1.414215...$$

$$X_3 = \varphi(X_2) = \frac{1}{2}\left(\frac{577}{408} + \frac{2}{577/408}\right) \cong 1.414213...$$

thus $\boxed{\sqrt{2} \cong 1.4142...}$ (got an extra digit as it happened)

(P103 & P104 are essentially on T3)

P105 $T(x,y) = (x+y,\ x-y,\ 3y)$

$$T(1,0) = (1,1,0)$$
$$T(0,1) = (1,-1,3) \Rightarrow \boxed{[T] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 3 \end{bmatrix}}$$

P106 $S(x,y,z) = x + 2y + 3z = [1,2,3]\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \boxed{[S] = [1,2,3]}$

P107 $(S \circ T)(x,y) = S(T(x,y))$

$$= S(x+y,\ x-y,\ 3y)$$
$$= x+y + 2(x-y) + 3(3y)$$
$$= 3x + 8y$$
$$= [3, 8]\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \boxed{[S \circ T] = [3, 8]}$$

Likewise $[S][T] = [1,2,3]\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 3 \end{bmatrix} = [1+2+0,\ 1-2+9] \overset{\checkmark}{=} [3, 8].$

P108 $F(x,y) = (x^2+y^2,\ xy)$

$$J_F = \left[\frac{\partial F}{\partial x} \,\middle|\, \frac{\partial F}{\partial y} \right] = \begin{bmatrix} 2x & 2y \\ y & x \end{bmatrix}$$

$$J_F(1,3) = \begin{bmatrix} 2 & 6 \\ 3 & 1 \end{bmatrix}, \quad F(1,3) = (1+9,\ 3) = (10, 3)$$

$$L_F^{(1,3)}(x,y) = (10,3) + \begin{bmatrix} 2 & 6 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x-1 \\ y-3 \end{bmatrix}$$

$$L_F^{(1,3)}(x,y) = (10 + 2(x-1) + 6(y-3),\ 3 + 3(x-1) + (y-3))$$

$$\boxed{L_F^{(1,3)}(x,y) = (-10 + 2x + 6y,\ 3x + y - 3)}$$

P109 Suppose $F: \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $x_0 \in \mathbb{R}^n$.

Then
$$\lim_{h \to 0} \underbrace{\left(\frac{F(x_0+h) - F(x_0) - dF_{x_0}(h)}{\|h\|} \right)}_{\eta(h)} = 0.$$

We have $\lim_{h \to 0} (\eta(h)) = 0$ and for $h \neq 0$,

$$F(x_0+h) - F(x_0) - dF_{x_0}(h) = \|h\| \eta(h)$$

Thus,
$$F(x_0+h) = F(x_0) + dF_{x_0}(h) + \|h\| \eta(h)$$

Notice $dF_{x_0}(h) = J_F(x_0)h$ thus $\lim_{h \to 0} (dF_{x_0}(h)) = \lim_{h \to 0} (J_F(x_0)h)$ by linearity of limit

$$= J_F(x_0) \lim_{h \to 0} (h)$$
$$= 0$$

Likewise, $\lim_{h \to 0} (\|h\| \eta(h)) = 0 \cdot 0 = 0.$

Therefore,
$$\lim_{h \to 0} (F(x_0+h)) = \lim_{h \to 0} \left(F(x_0) + \underbrace{dF_{x_0}(h)}_{0} + \overbrace{\|h\|}^{0} \overbrace{\eta(h)}^{0} \right)$$

$$\therefore \underline{\lim_{h \to 0} (F(x_0+h)) = F(x_0)} \quad //$$

P110 $F: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F(x,y) = (x^2 - y^2, 2xy)$

$$J_F = \left[\frac{\partial F}{\partial x} \middle| \frac{\partial F}{\partial y} \right] = \begin{bmatrix} 2x & -2y \\ 2y & 2x \end{bmatrix}$$

$$\det(J_F) = (2x)^2 + (2y)^2 = 4(x^2+y^2) \neq 0 \quad \text{for } (x,y) \neq (0,0).$$

P111 Find F^{-1} given $F(x,y) = (x^2-y^2,\ 2xy)$

$$\begin{aligned} x^2 - y^2 &= a \\ 2xy &= b \end{aligned} \quad \Rightarrow \quad y = \frac{b}{2x} \quad \Rightarrow \quad x^2 - \frac{b^2}{(2x)^2} = a$$

$$\Rightarrow x^2 - \frac{b^2}{4x^2} = a$$

$$\Rightarrow x^4 - ax^2 - \frac{b^2}{4} = 0$$

$$\Rightarrow \left(x^2 - \frac{a}{2}\right)^2 - \frac{a^2+b^2}{4} = 0$$

$$\Rightarrow x^2 - \frac{a}{2} = \pm\frac{1}{2}\sqrt{a^2+b^2}$$

$$\Rightarrow x^2 = \frac{a}{2} \pm \frac{1}{2}\sqrt{a^2+b^2}$$

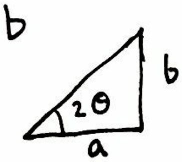
$$\Rightarrow x = \pm\sqrt{\frac{a}{2} \pm \frac{1}{2}\sqrt{a^2+b^2}}.$$

Then $y = \dfrac{b}{2x}$

$$y = \frac{\pm b}{2\sqrt{\frac{1}{2}\left[a \pm \sqrt{a^2+b^2}\right]}}.$$

(have to choose $\pm$ as appropriate for signs of a, b)

P112
$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}\right\} \quad \begin{aligned} x^2 - y^2 &= r^2(\cos^2\theta - \sin^2\theta) = r^2\cos(2\theta) = a \\ 2xy &= 2r^2\cos\theta\sin\theta = r^2\sin(2\theta) = b \end{aligned}$$



$$\frac{b}{a} = \frac{r^2\sin(2\theta)}{r^2\cos(2\theta)} = \tan(2\theta) \Rightarrow \theta = \frac{1}{2}\tan^{-1}\left(\frac{b}{a}\right)$$

$$a^2+b^2 = r^4\left(\cos^2(2\theta) + \sin^2(2\theta)\right) = r^4 \quad \therefore \quad r = \sqrt[4]{a^2+b^2}$$

$$\boxed{\begin{aligned} x &= \sqrt[4]{a^2+b^2}\cos\left(\tfrac{1}{2}\tan^{-1}\left(\tfrac{b}{a}\right)\right) \\ y &= \sqrt[4]{a^2+b^2}\sin\left(\tfrac{1}{2}\tan^{-1}\left(\tfrac{b}{a}\right)\right) \end{aligned}}$$

$\boxed{P112}$ continued, another way to look at it if you know complex variables $z = x+iy$

$$z^2 = (x+iy)(x+iy) = x^2 - y^2 + 2ixy$$

If $z^2 = w$ then $z = \sqrt{w}$

But, in complex variables $\sqrt{w} = \sqrt{|w|e^{i\beta}} = \sqrt{|w|}\exp(i\beta/2)$

Here $w = a+ib$ and $|w| = \sqrt{a^2+b^2}$ and β is standard angle of (a,b) thus $\beta = \tan^{-1}(b/a)$ for $a>0$ etc.

$$z = \sqrt{w}$$

$$x+iy = \sqrt{\sqrt{a^2+b^2}}\,\big(\overbrace{\cos(\beta/2) + i\sin(\beta/2)}^{e^{i\beta/2}}\big)$$

$$x+iy = (a^2+b^2)^{1/4}\left[\cos\left(\tfrac{1}{2}\tan^{-1}\left(\tfrac{b}{a}\right)\right) + i\sin\left(\tfrac{1}{2}\tan^{-1}\left(\tfrac{b}{a}\right)\right)\right]$$

which is $\approx$ to what I found w/o complex notation.

$\boxed{P113}$ $F(x,y,z) = (x/y,\ y/z,\ z/x)$

$$J_F = \left[\frac{\partial F}{\partial x}\Big|\frac{\partial F}{\partial y}\Big|\frac{\partial F}{\partial z}\right] = \begin{bmatrix} 1/y & -x/y^2 & 0 \\ 0 & 1/z & -y/z^2 \\ -z/x^2 & 0 & 1/x \end{bmatrix}$$

$$\det(J_F) = \frac{1}{y}\det\begin{bmatrix} 1/z & -y/z^2 \\ 0 & 1/x \end{bmatrix} + \frac{x}{y^2}\det\begin{bmatrix} 0 & -y/z^2 \\ -z/x^2 & 1/x \end{bmatrix}$$

$$= \frac{1}{y}\left[\frac{1}{z}\frac{1}{x}\right] - \frac{x}{y^2}\left[\frac{z}{x^2}\right]\left[\frac{y}{z^2}\right]$$

$$= 0$$

Every point in dom(F) is singular.

$\boxed{P114}$ $a = \frac{x}{y}$, $b = \frac{y}{z}$, $c = \frac{z}{x}$ $\Rightarrow$ $abc = \frac{x}{y}\frac{y}{z}\frac{z}{x} = 1$

thus $c = \frac{1}{ab}$, but a, b, c should be independent...

P114 continued (I'm just exploring why F^{-1} does not exist, you can skip this if you want)

We found J_F is everywhere singular for $F(x,y,z) = (x/y,\ y/z,\ z/x)$. This makes me suspect F is nowhere invertible.

Notice $dom(F) = \{(x,y,z) \in \mathbb{R}^3 \mid x,y,z \neq 0\}$

Let $(x,y,z) \in dom(F)$ and note $c(x,y,z) \in dom(F)$ for $c \neq 0$

In particular, we can choose c small enough so (x,y,z) and $c(x,y,z) = (cx, cy, cz)$ are very close (oh, $c = 1+\varepsilon$, $\varepsilon > 0$ so c not so much small as it is close to 1)

But,

$$\begin{aligned} F(x,y,z) &= (x/y,\ y/z,\ z/x) \\ &= (cx/cy,\ cy/cz,\ cz/cx) \\ &= F(cx, cy, cz) \end{aligned}$$

So, F is not 1-1 on the set containing (x,y,z), (cx,cy,cz) and this implies F is not injective on any open set containing (x,y,z). More to the point, if $abc = 1$ then

$$\begin{aligned} F^{-1}\{(a,b,c)\} &= \{(x,y,z) \mid \underbrace{a = \tfrac{x}{y},\ b = \tfrac{y}{z},\ c = \tfrac{z}{x}}\} \\ &= \{(x, bcx, cx) \mid x \neq 0\} \end{aligned}$$

$x = ay$, $y = bz$, $z = cx$

$x = abz = abcx \Rightarrow x = x$ as $abc = 1$.

$y = bz = bcx$

$z = cx$

and if $abc \neq 1$ then

$$F^{-1}\{(a,b,c)\} = \emptyset.$$

P115 Find multivariate power series centered at $(0,0)$

$$f(x,y) = x\sin(y) + \cos(x)$$
$$= x\left(y - \frac{1}{6}y^3 + \frac{1}{120}y^5 + \cdots\right) + 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \cdots$$
$$= \boxed{1 + xy - \frac{1}{2}x^2 - \frac{1}{6}xy^3 + \frac{1}{24}x^4 + \frac{1}{120}xy^5 - \frac{1}{6!}x^6 + \cdots}$$

P116 Find multivariate power series centered at $(1,-1)$ for

$$f(x,y) = e^{xy}$$

$$f_x = ye^{xy}$$
$$f_y = xe^{xy}$$
$$f_{xx} = y^2e^{xy}$$
$$f_{yy} = x^2e^{xy}$$
$$f_{xy} = e^{xy} + xye^{xy} = (1+xy)e^{xy}$$

Then, to 2^{nd} order,

$$f(x,y) = f(1,-1) + f_x(1,-1)(x-1) + f_y(1,-1)(y+1) + \frac{1}{2}f_{xx}(1,-1)(x-1)^2 + f_{xy}(1,-1)(x-1)(y+1) + \frac{1}{2}f_{yy}(1,-1)(y+1)^2 + \cdots$$

Hence, factoring out e^{-1} from all terms,

$$\boxed{e^{xy} = e^{-1}\left(1 - (x-1) + (y+1) + \frac{1}{2}(x-1)^2 + \frac{1}{2}(y+1)^2 + \cdots\right)}$$

Another way is to do same algebra with substitution for exponential series $e^u = 1 + u + \frac{1}{2}u^2 + \frac{1}{3!}u^3 + \cdots$

$$e^{xy} = e^{(x-1+1)(y+1-1)}$$
$$= e^{(x-1)(y+1) + (y+1) - (x-1) - 1}$$
$$= e^{-1}\left[e^{(x-1)(y+1)}e^{(y+1)}e^{-(x-1)}\right]$$
$$= e^{-1}\left[1 + (x-1)(y+1) + \frac{1}{2}(x-1)^2(y+1)^2 + \cdots\right]\left[1 + (y+1) + \frac{1}{2}(y+1)^2 + \cdots\right] \cdot \left[1 - (x-1) + \frac{1}{2}(x-1)^2 + \cdots\right]$$
$$= \underline{e^{-1}\left[1 + (y+1) - (x-1) + \frac{1}{2}(x-1)^2 + \frac{1}{2}(y+1)^2 + \cdots\right]}.$$

(the $(x-1)(y+1)$ terms cancel out)

$\boxed{\text{P117}}$ $Q(x,y) = 10x^2 - 8xy + 10y^2$

$[Q] = \begin{bmatrix} 10 & -4 \\ -4 & 10 \end{bmatrix}$ has $Q(x,y) = [x, y]\begin{bmatrix} 10 & -4 \\ -4 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$.

Eigenvalues are sol^{n}'s of $\det([Q] - \lambda I) = 0$ $\left(I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$

$$\det\begin{bmatrix} 10-\lambda & -4 \\ -4 & 10-\lambda \end{bmatrix} = (10-\lambda)^2 - 16$$
$$= (\lambda - 10)^2 - 4^2$$
$$= (\lambda - 14)(\lambda - 6) \Rightarrow \underline{\lambda_1 = 6 \ \& \ \lambda_2 = 14}$$

Therefore, the formula for Q in eigencoordinates $\bar{x}_1, \bar{x}_2$ w.r.t. an orthonormal eigenbasis $\{u_1, u_2\}$ where $Au_1 = 6u_1$ & $Au_2 = 14u_2$ is simply,

$$\boxed{Q(x,y) = 6\bar{x}_1^2 + 14\bar{x}_2^2}$$

Then $Q(x,y) = 1 \Rightarrow 6\bar{x}_1^2 + 14\bar{x}_2^2 = 1$ the curve is an

$\Rightarrow \boxed{\dfrac{\bar{x}_1^2}{1/6} + \dfrac{\bar{x}_2^2}{1/14} = 1}$ $\underline{\text{ellipse}}$

$\boxed{\text{P118}}$ $Q(x,y,z) = 31x^2 + 15y^2 + 15z^2 - 22xy + 10yz$

$$\Rightarrow [Q] = \begin{bmatrix} 31 & -11 & 0 \\ -11 & 15 & 5 \\ 0 & 5 & 15 \end{bmatrix} \Rightarrow \left.\begin{matrix} \lambda_1 \cong 6.9 \\ \lambda_2 \cong 17.3 \\ \lambda_3 \cong 36.9 \end{matrix}\right\}$$ using the website I shared.

arndt-bruenner.de

$\therefore$ $Q(x,y,z) \cong 6.9\bar{x}_1^2 + 17.3\bar{x}_2^2 + 36.9\bar{x}_3^2$ in eigencoordinates.

thus $Q(x,y,z) = 1 \longleftrightarrow \boxed{\underbrace{6.9\bar{x}_1^2 + 17.3\bar{x}_2^2 + 36.9\bar{x}_3^2 = 1}_{\text{ellipsoid}}}$

$\boxed{P119}$ $f(x,y) = 3 + 10(x-1)^2 - 4(x-1)(y-2) + 6(y-2)^2 + \cdots$

Compare against for $P = (1,2)$

$$f(x,y) = f(P) + f_x(P)(x-1) + f_y(P)(y-2) + \underbrace{\tfrac{1}{2}f_{xx}(P)(x-1)^2 + f_{xy}(P)(x-1)(y-2) + \tfrac{1}{2}f_{yy}(P)(y-2)^2}_{Q(x-1,\,y-2)} + \cdots$$

to see that

$$f(1,2) = 3$$
$$f_x(1,2) = f_y(1,2) = 0$$
$$f_{xx}(1,2) = 20$$
$$f_{xy}(1,2) = -4$$
$$f_{yy}(1,2) = 12$$

Thus $\nabla f(1,2) = \langle 0,0\rangle$ $\therefore$ $(1,2)$ is critical point.

Moreover $[Q] = \begin{bmatrix} 20 & -2 \\ -2 & 12 \end{bmatrix}$ hence

$$\det([Q] - \lambda I) = \det\begin{bmatrix} 20-\lambda & -2 \\ -2 & 12-\lambda \end{bmatrix}$$
$$= (20-\lambda)(12-\lambda) - 4$$
$$= \lambda^2 - 32\lambda + 240 - 4$$
$$= \lambda^2 - 32\lambda + 236 = 0 \Rightarrow$$

(I used website to check these)

$\lambda_1 \approx 11.5$

$\lambda_2 \approx 20.5$

Thus $f(x,y) = 3 + 11.5\bar{x}_1^2 + 20.5\bar{x}_2^2 + \cdots$

imply $f(1,2) = 3$ is local minimum

Th^m/ At critical point

- if $\lambda_1, \lambda_2, \ldots, \lambda_n > 0$ then $f(P)$ is local min
- if $\lambda_1, \lambda_2, \ldots, \lambda_n < 0$ then $f(P)$ is local max.
- if $\lambda_1, \lambda_2, \ldots, \lambda_n$ have both positive & negative values then $f(P)$ is saddle.
- if any e-value is zero then further analysis is required.

$\boxed{P120}$

$$f(x,y,z) = \frac{1+y^2}{1-2xz} \quad : \quad \frac{1}{1-2xz} = \frac{1}{1-r} = 1 + r + r^2 + \cdots \quad \text{for } r = 2xz$$

$$= (1+y^2)(1 + 2xz + 4x^2z^2 + \cdots)$$

$$= 1 + y^2 + 2xz + \cdots \quad \Rightarrow \quad \nabla f(0,0,0) = \langle 0,0,0 \rangle$$

since no linear terms.

Of course, you could also just calculate f_x, f_y, f_z and see they evaluate to 0 at $(0,0,0)$. Next, comparing the power series expansion I found against,

$$f(x,y,z) = f(0) + f_x(0)x + f_y(0)y + f_z(0)z + \frac{1}{2}f_{xx}(0)x^2 + \frac{1}{2}f_{yy}(0)y^2 + \frac{1}{2}f_{zz}(0)z^2 + f_{xy}(0)xy + f_{xz}(0)xz + f_{yz}(0)yz + \cdots$$

Reveals

$$[Q] = \begin{bmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{xy} & f_{yy} & f_{yz} \\ f_{xz} & f_{yz} & f_{zz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Then, I'll be lazy and use website again to find e-values for Hessian $[Q]$ are

$$\lambda_1 = -1, \quad \lambda_2 = \lambda_3 = 1$$

Thus $(0,0,0)$ is saddle-type (it's neither minimum nor maximum near $(0,0,0)$)