MH 408: Topics in Analysis: Test 1

Show work for full credit. A scientific, non-graphing, calculator is allowed. You are also allowed one page of notes on regular sized paper front and back. At least 100pts to earn here.

(1.) (10pts) Let A, B, C, D be sets. Suppose $A \subseteq B$ and $C \subseteq D$. Prove $A \times C \subseteq B \times D$.

(2.) Suppose $x, y \in \mathbb{Z}$. Define $x \sim y$ if and only if there exists $j \in \mathbb{Z}$ for which $y - x = 4j$.

(a.) (12pts) Prove that $\sim$ defines an equivalence relation on $\mathbb{Z}$.

(b.) (5pts) List all the equivalence classes for $\sim$.

(3.) (15pts) Let $f(x) = \dfrac{x}{x-2}$ for $x \in \mathbb{R}$ with $x \neq 2$. Find a set $B \subseteq \mathbb{R}$ for which $f : (-\infty, 2) \cup (2, \infty) \to B$ is a bijection. Once you make your choice of B, then prove f is a bijection.

(4.) (18pts) Let $f(x) = 3 + (x-4)^2$. Find:

(a.) $f([0,1])$

(b.) $f^{-1}([0,1])$

(c.) $f^{-1}([3,4])$

(5.) (15pts) Let $f : X \to Y$ be a function. Suppose $A, B \subseteq X$. Prove $A \subseteq B$ implies $f(A) \subseteq f(B)$.

(6.) (15pts) Prove $1^3 + 2^3 + \cdots + n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$ for all $n \in \mathbb{N}$.

(7.) (20pt) Let $S = \{\frac{2n}{n+3} \mid n \in \mathbb{N}\}$. Find $\inf(S)$ and $\sup(S)$. In addition, justify your claims.