MH 408: Topics in Analysis:	Test 2

Show work for full credit. A scientific, non-graphing, calculator is allowed. You are also allowed one page of notes on regular sized paper front and back. At least 100pts to earn here.

(1.) (10pts) Use limit laws and/or the appropriate theorem to show that

$$\lim_{n\to\infty}\left(\frac{n+\sin(n^4+4n+17)}{3n^2+7}\right)=0.$$

(2.) (10pts) Prove $a_n = \cos\left(\frac{n\pi}{3}\right)$ is a divergence sequence.

(3.) (10pts) Prove $a_n = \frac{n\sin(4n^2+2)}{n+3}$ has a convergent subsequence.

(4.) (10pts) Prove $\lim_{x\to 2}\left(\frac{x+13}{7-x}\right) = 3$ by direct argument based on the $\varepsilon-\delta$ definition of the limit.

(5.) (10pts) Let $f(x) = 2x^2+3$. Prove f is continuous on $\mathbb{R}$ by direct argument from the $\varepsilon-\delta$-definition of continuity.

(6.) (10pts) Let $f : D \to \mathbb{R}$ and $g : D \to \mathbb{R}$ be functions which are continuous at $x_o \in D$. Prove $f+g$ is continuous at $x_o \in D$ by a by direct argument from the $\varepsilon-\delta$-definition of continuity.

(7.) (10pts) Suppose A and B are compact subsets of $\mathbb{R}$. Prove $A\cap B$ is compact.

(8.) (10pts) Fix $a, b, c \in \mathbb{R}$. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^3 + ax^2 + bx + c$. Prove there exists $x_o \in \mathbb{R}$ for which $f(x_o) = 0$. You may assume it is known that f is continuous.

(9.) (10pts) Let $f : [a, b] \to [2a, 2b]$ be a continuous function. Prove $f(x) = 2x$ has a solution on $[a, b]$.

(10.) (10pts) Suppose f, g are continuous functions on $\mathbb{R}$ and $f(x) = g(x)$ for all $x \in \mathbb{R}-\mathbb{Q}$. Prove $f(x) = g(x)$ for all $x \in \mathbb{R}$. You may assume it is known that for each $x \in \mathbb{Q}$ there exists a sequence $\{x_n\}$ such that $x_n \in \mathbb{R}-\mathbb{Q}$ for all $n \in \mathbb{N}$ and $x_n \to x$.

(11.) (10pts) Choose to answer **one** of the following:

- **(a.)** (10pts) Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is nowhere continuous.
- **(b.)** (10pts) Give an example of a continuous function whose nonempty domain has no limit points.