MH 408: TOPICS IN ANALYSIS:	TEST 3

Show work for full credit. At least 100pts to earn here.

(1.) (10pts) Prove $\frac{d}{dx}\cos x = -\sin x$ given that $\lim_{h\to 0}\frac{\sin(h)}{h} = 1$ and $\lim_{h\to 0}\frac{1-\cos(h)}{h} = 0$.

(2.) (10pts) Exercise 4.2.1 from the text.

(3.) (10pts) Use the Mean Value Theorem to prove $e^x > 1 + x$ for $x > 0$. Assume we know $\frac{d}{dx}e^x = e^x$.

(4.) (10pts) Exercise 4.3.5 from the text.

(5.) (10pts) Exercise 4.5.3 from the text.

(6.) (10pts) Prove that the equation $2 - x - \sin x = 0$ has a unique real root, and that it lies in the interval $[\pi/6, \pi/2]$. Show that $\phi(x) = 2 - \sin(x)$ is a contraction mapping of this interval.

(7.) (10pts) Let $T : \mathbb{R}^4 \to \mathbb{R}^2$ be defined by $T(a, b, c, d) = (a + 2b + 3c, 4b + 5c + 6d)$ for each $(a, b, c, d) \in \mathbb{R}^4$. Find the standard matrix for T.

(8.) (15pts) Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $F(x, y, z) = (x^2 + y^2 + z^2, xyz, x + y + z)$.
Find the linearization of F at (x_o, y_o, z_o). At which points does the inverse mapping theorem indicate the existence of a local inverse for F ?

(9.) (15pts) Consider the quadratic form $Q(x, y) = 4x^2 + 2xy + 4y^2$. Find the eigenvalues of the quadratic form. Write the equation on $Q(x, y) = 1$ in eigencoordinates and identify the curve.

(10.) (10pts) Let $f(x, y, z) = 4 + 4(x-1)^2 + 2(x-1)(y-2) + 4(y-2)^2 + 4(z-3)^2 + \cdots$. Determine if $f(1, 2, 3) = 4$ is a minimum, maximum or a saddle point in view of the given multivariate Taylor expansion.