

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on this cover sheet. Work supporting your answers is to be given on additional sheets past this cover sheet. Please number your sheets and create a single pdf which legibly records your work. Thanks!

Problem 1 Your PRINTED NAME below indicates you have:

(a.) I have read §1.1 – 1.4 of Cook: _____.

Problem 2 Find the distance from $P = (1, 2, 3)$ to:

- (a) the point $Q = (0, -1, 7)$
- (b) the x -axis
- (c) the line through $(1, 10, 3)$ and $(2, 2, 2)$

Problem 3 Let $\vec{A} = \langle 1, 1, 4 \rangle$ and $\vec{B} = \langle 0, 3, 4 \rangle$. Calculate the following:

- (a) A and B
- (b) \hat{A} and \hat{B}
- (c) $\vec{A} \cdot \vec{B}$
- (d) $\vec{A} \times \vec{B}$
- (e) the angle between \vec{A} and \vec{B}
- (f) the area of the parallelogram with sides \vec{A}, \vec{B} .
- (g) write $\vec{A} = \vec{v}_1 + \vec{v}_2$ where \vec{v}_1 is parallel to \vec{B} and \vec{v}_2 is perpendicular to \vec{B} .

Problem 4 Let \vec{V} be a vector which makes an angle of 120° with the positive x -axis, 60° with the positive y -axis and 45° with the positive z -axis. In addition, you are given $\vec{V} \cdot \hat{x} = -10$. Find \vec{V} .

Problem 5 Suppose P, Q, R are the vertices of a triangle. Furthermore, suppose a, b are constants with $a > 0$ and $P = (-1, 1, a)$ and $Q = (1, b, 3)$ and R is at the origin. Also, you are given the interior angle at R is 90° . Finally, you are also given that P is a distance of $\sqrt{102}$ from R . Find a, b and find the angle interior to the triangle at P and Q .

Problem 6 Show that $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$.

Problem 7 Suppose \vec{A}, \vec{B} are perpendicular vectors. Use dot-products to show that

$$\|\vec{A} + \vec{B}\|^2 = \|\vec{A}\|^2 + \|\vec{B}\|^2.$$

Problem 8 Let $\vec{A} = \langle 2, 2, 1 \rangle$ and $\vec{B} = \langle 1, 1, -4 \rangle$. Find \vec{C} with length 1 which is orthogonal to both \vec{A} and \vec{B} . Then, find c_1, c_2, c_3 for which $\langle 3, 4, 5 \rangle = c_1\vec{A} + c_2\vec{B} + c_3\vec{C}$.

Problem 9 Let a, b, c, d be constants. Find k for which $(a\vec{A} + b\vec{B}) \times (c\vec{A} + d\vec{B}) = k\vec{A} \times \vec{B}$.

Problem 10 Calculate $[\hat{x} \times \langle a, b, c \rangle] \times \hat{x}$ and $[\hat{y} \times \langle a, b, c \rangle] \times \hat{y}$. Conjecture the result of $[\hat{z} \times \langle a, b, c \rangle] \times \hat{z}$.

Problem 11 Let \hat{u} be a unit-vector. Let \vec{A} be an arbitrary vector. Show that:

$$\vec{A} = [\vec{A} \cdot \hat{u}] \hat{u} + [\hat{u} \times \vec{A}] \times \hat{u}.$$

Then, identify the given formulas with proj and orth operations as discussed in my notes (Definition 1.1.24). The identity $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$ is helpful here.

Problem 12 Suppose $\vec{v} = \langle v_x, v_y \rangle$. Find the standard angle and magnitude of \vec{v} if

- (a) $v_x = 1$ and $v_y = 2$
- (b) $v_x = -1$ and $v_y = -2$
- (c) $v_x = 0$ and $v_y = -3$

Problem 13 Suppose $\vec{r}(t) = \langle 2 + 3t, 3 + t, 4 - 2t \rangle$ is the parametrization of a line which lies in a plane which also contains the point $(1, 4, 8)$. Find the Cartesian equation of this plane. Finally, if a ninja is at $(10, 14, 4)$ then how far is the ninja off the plane?

Problem 14 Suppose a line goes through $P = (2, 3, 4)$ and $Q = (3, 3, -3)$.

- (a) find a parametrization $\vec{r}(t)$ of the line for which $t = 0$ corresponds to $(2, 3, 4)$
- (b) find a parametrization $\vec{R}(t)$ of the line for which $t = 0$ corresponds to the midpoint of P and Q
- (c) find the Cartesian equations of the line.

Problem 15 Consider the points $P = (1, 0, 1)$, $Q = (2, 3, 3)$ and $R = (5, 6, 0)$. Consider the parallelogram with sides \overline{PQ} and \overline{PR} .

- (a) find a parametrization $\vec{r}(s, t)$ of the parallelogram where $0 \leq s, t \leq 1$
- (b) find the Cartesian equation of the plane which contains the parallelogram.
- (c) write the parallelogram as a graph $z = f(x, y)$ and be sure to explicitly find the appropriate domain for f .
- (d) is the point $(5/2, 9/2, 1/2)$ in the parallelogram?

Bonus: prove $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$ using index calculation.