Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on this cover sheet. Work supporting your answers is to be given on additional sheets past this cover sheet. Please number your sheets and create a single pdf which legibly records your work. Thanks!

Problem 46 Your PRINTED NAME below indicates you have:

(a.) I have read Chapter 4 of Cook: ____________________.

Problem 47 Suppose Paccun speeds towards the base of a valley with shape given by the equation $z = 5x^2 + xy + 3y^2$ where x, y, z are in meters.

(a) What is the direction of steepest **descent** at the point $(1, -1, 7)$?

(b) Assuming Paccun stays on the surface of the valley, if $dx/dt = 3\,m/s$ and $dy/dt = 4m/s$ then what is Paccun's speed[1] ?

Problem 48 Label the solution set of $y^2 = x - z^2$ as M.

(a.) present M as a level-surface for some function F. Explicitly state the formula for F. Find the normal vector field on M.

(b.) Parametrize M for $x > 0$ by using $x = t^2$ and $y = t\cos\beta$. Find the parametrization of z and then calculate $\vec{N}$ explicitly in terms of the given parameters.

Problem 49 Suppose $z = e^{x^2} + \sin(xy)$ and $x = \exp[g(t)]$ and $y = h(t^2 + 1)$ for some differentiable functions g, h. Calculate dz/dt by the chain rule(s). Your answer will have the unknown functions h and g.

Problem 50 Let $w = z^2 + \exp(x^3 + y^4)$ and $x = 2st$, $y = s^2 - t^2$ and $z = s + t$. Calculate $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$. You may use the intermediate variables x, y and z in the formulation of your answer.

Problem 51 Let $S_1 : x^2 + y^2 + z^2 = 4$ and $S_2 : x^2 + y^2 = 2$. If C is formed by the intersection of S_1 and S_2 then parametrize C and relate the direction of the normal to S_1 and the normal of S_2 to the direction of C (at points of C naturally).

Problem 52 Let $w = x^2 + y^3 + z^4$. Calculate $\frac{\partial w}{\partial x}$ given that $y^2 = x^3 + e^z$ in the following cases:

(a) using x, y as independent variables; in precise notation, calculate $\left(\frac{\partial w}{\partial x}\right)_y$,

(b) using x, z as independent variables; in precise notation, calculate $\left(\frac{\partial w}{\partial x}\right)_z$

Problem 53 You are given $dz + dw = x^2dx + xydy$ and $dz - dw = e^x dx + \sin(x + w)dy$. Calculate $\left(\frac{\partial z}{\partial x}\right)_y$. You may leave the answer in terms of x, y and w despite the fact that w is viewed as a dependent variable in this calculation.

[1] recall, speed is defined as the magnitude of the velocity

Problem 54 The following problems are closely related:

(a) If $\vec{A}(x,y) = \langle 1+e^y,\ xe^y + y^2 \rangle$ then decide if there exists f such that $\nabla f = \vec{A}$. If there does indeed exists such f then calculate its formula.

(b) Solve $(1+e^y)\,dx + (\ln x - 2)\,dy = 0$.

Problem 55 Suppose $x^2 + 3xyz + z^3 = 10$. Show that $\frac{\partial z}{\partial y}\frac{\partial y}{\partial x}\frac{\partial x}{\partial z} = -1$. Notice, to understand this we must consider x, y, z both as dependent and independent to complete the calculation. Also, explain where the implicit function theorem allows us to view z locally as a graph of a function in x, y. What about y? Where can we view $y = y(x,z)$?

Problem 56 Fun with non-cartesian coordinate formulae: (these are only in my notes)

(a) Let $g(r,\theta) = r^2\theta^2$ calculate ∇g in terms of the polar coordinate frame; that is, express ∇g in terms of functions of r and θ as well as $\widehat{r}$ and $\widehat{\theta}$.

(b) Suppose $\vec{F} = \widehat{\rho}\rho^2 + \widehat{\phi}\frac{\cos\phi}{\rho} + \widehat{\theta}\frac{\theta e^\theta}{\rho\sin\phi}$. Find f such that $\nabla f = \vec{F}$.

Problem 57 Let $f : \mathbb{R}^n \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be smooth functions then the chain-rule combines first semester differentation and the gradient as follows: $\nabla h(f(\vec{r})) = h'(f(\vec{r}))\nabla f(\vec{r})$ where $\vec{r} = (x_1, x_2, \ldots, x_n)$. Let $r = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$ and calculate the following with the help of the chain-rule just given:

(a) ∇r

(b) ∇r^2

(c) $\nabla(1/r)$

Problem 58 Let $u = 3x + 2y$ and $v = x + y$ define new variables for $\mathbb{R}^2$. Convert Laplace's equation $\phi_{xx} + \phi_{yy} = 0$ to the (u,v) coordinate system. It may be helpful to consider the chain rules in operator notation:

$$\frac{\partial}{\partial x} = \frac{\partial u}{\partial x}\frac{\partial}{\partial u} + \frac{\partial v}{\partial x}\frac{\partial}{\partial v} \qquad \& \qquad \frac{\partial}{\partial y} = \frac{\partial u}{\partial y}\frac{\partial}{\partial u} + \frac{\partial v}{\partial y}\frac{\partial}{\partial v}.$$

Problem 59 Show that if f is differentiable at each point of the line segment $\overline{PQ}$ and $f(P) = f(Q)$ then there exists a point R between P and Q for which $\nabla f(R)$ is orthogonal to $\overrightarrow{PQ}$.

Problem 60 Observe $F(x,y,z,t)$ has total differential $dF = (\partial_x F)dx + (\partial_y F)dy + (\partial_z F)dz + (\partial_t F)dt$. We say $Pdx + Qdy + Rdz + Sdt = 0$ is exact if there exists F for which $dF = Pdx + Qdy + Rdz + Sdt$. What 6 conditions must be met in order that $Pdx + Qdy + Rdz + Sdt = 0$ is an exact equation? Is $ydx + xdy + z^3dz + (t+x)dt = 0$ exact ?