

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on this cover sheet. Work supporting your answers is to be given on additional sheets past this cover sheet. Please number your sheets and create a single pdf which legibly records your work. Thanks!

Problem 76 Your PRINTED NAME below indicates you have:

(a.) I have read Chapter 6 of Cook: _____.

Problem 77 Calculate $\int_0^1 \int_x^{x^3} y^2 dy dx$

Problem 78 Calculate $\int_0^1 \int_0^x \int_{x^2+y^2}^{x^3+y^3} 2z dz dy dx$

Problem 79 Let $f(x, y) = x^2 + y^3$. Find the average of f on the square $[0, 1] \times [0, 1]$.

Problem 80 Reverse the order of integration in order to calculate the following integral:

$$\int_0^1 \int_y^1 \frac{2}{1+x^4} dx dy.$$

Problem 81 Let B be the solid region bounded by $x = 0$, $y = 0$, $z = 0$ and the plane $2x + 2y - 3z = 1$. Calculate the volume of B .

Problem 82 Let R be the regions in quadrant I bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ and $y = x$ and $y = 3x$. Calculate $\iint_R \sqrt{x^2 + y^2} dA$ by changing the integration to polar coordinates.

Problem 83 Find the volume bounded by the cylinder $x^2 + y^2 = 1$ and $z = 2 + x + y$ and $z = 1$.

Problem 84 Let B be a ball of radius R centered at the origin. Calculate $\iiint_B e^{-\rho^3} dV$.

Problem 85 Find the centroid of the region bounded by $r = 4 \cos \theta$ and $r = 2$.

Problem 86 Suppose the density of a solid sphere B of radius R is given by $\delta = \alpha \rho^c$ where α, c are constants and $\rho = \sqrt{x^2 + y^2 + z^2}$. The units of the constants are chosen as to give $\delta = \frac{dm}{dV}$ units of mass per volume. The moment of inertia with respect to the z axis is given by $I = \iiint_B r^2 \delta dV$ where $r^2 = x^2 + y^2$. Calculate I as a function of the constants c and α .

Problem 87 Calculate $\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (12 - x^2 - y^2 - \sqrt{x^2 + y^2}) dy dx$ by changing to an integral in polar coordinates.

Problem 88 Let P be the parallelogram with vertices $(0, 0)$, $(0, 1)$, $(2, 4)$, $(2, 5)$. Calculate

$$\iint_P \sqrt{2x(y - 2x)} \, dA$$

by making a substitution of $x = 2u$ and $y - 2x = v$. Notice, this substitution changes P to $[0, 1] \times [0, 1]$ in u, v space. Thanks to Brigg's and Cochran section 14.7 for this example.

Problem 89 Use the Jacobian to relate the volume element $du \, d\phi \, d\theta$ to the Cartesian volume element $dx \, dy \, dz$ given that

$$x = u \cos \theta \cosh \phi, \quad y = u \sin \theta \cosh \phi, \quad z = u \sinh \phi.$$

Problem 90 Calculate the hypervolume of the set $\{(x, y, z, t) \mid x^2 + y^2 + z^2 + t^2 \leq 1\}$