

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on this cover sheet. Work supporting your answers is to be given on additional sheets past this cover sheet. Please number your sheets and create a single pdf which legibly records your work. Thanks!

**Problem 106** Your PRINTED NAME below indicates you have:

(a.) I have read §7.6 – 7.10 of Cook: \_\_\_\_\_.

**Problem 107** Find the surface area of torus with radii  $A, R > 0$  and  $R \geq A$  parametrized by

$$\vec{X}(\alpha, \beta) = \left\langle [R + A \cos(\alpha)] \cos(\beta), [R + A \cos(\alpha)] \sin(\beta), A \sin(\alpha) \right\rangle$$

for  $0 \leq \alpha \leq 2\pi$  and  $0 \leq \beta \leq 2\pi$ .

**Problem 108** Let  $S$  be the outward oriented unit-sphere. Calculate  $\iint_S \langle x^3, y^3, z^3 \rangle \cdot d\vec{S}$ .

**Problem 109** Consider a thin-shell of constant density  $\delta$ . Let the shell be cut from the cone  $x^2 + y^2 - z^2 = 0$  by the planes  $z = 1$  and  $z = 2$ . Find (a.) the center of mass and (b.) the moment of inertia with respect to the  $z$ -axis.

**Problem 110** Find the flux of  $\vec{F}(x, y, z) = \langle z^2, x, -3z \rangle$  through the parabolic cylinder  $z = 4 - y^2$  bounded by the planes  $x = 0$ ,  $x = 1$  and  $z = 0$ . Assume the orientation of the surface is outward, away from the  $x$ -axis.

**Problem 111** Find the flux of  $\vec{F}(x, y, z) = \langle -x, -y, z^2 \rangle$  through the conical frustrum  $z = \sqrt{x^2 + y^2}$  between the planes  $z = 1$  and  $z = 2$  with outward orientation.

**Problem 112** Suppose  $\vec{C}$  is a constant vector. Let  $\vec{F}(x, y, z) = \vec{C}$  find the flux of  $\vec{F}$  through a surface  $S$  on plane with nonzero vectors  $\vec{A}, \vec{B}$ . In particular, the surface  $S$  is parametrized by  $\vec{r}(u, v) = \vec{r}_o + u\vec{A} + v\vec{B}$  for  $(u, v) \in \Omega$ .

**Problem 113** Let  $\phi = \pi/4$  define a closed surface  $S$  with  $0 \leq \rho \leq 2$ . Find the flux of

$$\vec{F}(\rho, \phi, \theta) = \phi^2 \hat{\rho} + \rho \hat{\phi} + \hat{\theta}$$

through the outward oriented  $S$ .

**Problem 114** Consider the closed cylinder  $x^2 + y^2 = R^2$  for  $0 \leq z \leq L$ . Find the flux of

$$\vec{F}(r, \theta, z) = \theta \hat{z} + z \hat{\theta} + r^2 \hat{r}$$

out of the cylinder.

**Problem 115** Let  $\vec{F}(x, y, z) = \langle y^2 + z^2, x^2 + z^2, x^2 + y^2 \rangle$ . Find the work done by  $\vec{F}$  around the CCW (as viewed from above) triangle formed from the intersection of the plane  $x + y + z = 1$  and the coordinate planes. (use Stoke's Theorem)

**Problem 116** Let  $\vec{F} = \langle 2x, 2y, 2z \rangle$  and suppose  $S$  is a simply connected surface with boundary  $\partial S$  a simple closed curve. Give two arguments (one by Stokes', the other by Gauss' theorem) that  $\int_{\partial S} \vec{F} \cdot d\vec{r} = 0$ .

**Problem 117** Suppose  $S$  is the union of the cylinder  $x^2 + y^2 = 1$  for  $0 \leq z \leq 1$  and the disk  $x^2 + y^2 \leq 1$  at  $z = 1$ . Suppose  $\vec{F}$  is a vector field such that

$$\nabla \times \vec{F} = \left\langle \sinh(z)(x^2 + y^2), ze^{xy + \cos(x+y)}, (xz + y) \tan^{-1}(z) \right\rangle.$$

Calculate the flux of  $\nabla \times \vec{F}$  through  $S$ .

**Problem 118** Let  $E$  be the cube  $[-1, 1]^3$ . Calculate the flux through  $\partial E$  of the vector field

$$\vec{F}(x, y, z) = \langle y - x, z - y, y - x \rangle$$

(please use the divergence theorem!)

**Problem 119** Calculate the flux of  $\vec{F}(x, y, z) = \langle 1, 2, z^2 \rangle$  out of the paraboloid  $z = 4 - x^2 - y^2$  bounded below by the  $xy$ -plane.

**Problem 120** Let a spherical shell  $S$  of radius  $R$  and total mass  $M$  have constant mass density  $\sigma = \frac{dm}{dS}$ . Find the moment of inertia for this shell with respect to the  $z$ -axis. In particular, calculate

$$\iint_S \sigma r^2 dS.$$