Please box your answer. Show work for full credit. A scientific, non-graphing, calculator is allowed. You are also allowed one page of notes on regular sized paper front and back. Please give angles in degrees. Notation: for $\vec{A} \neq 0$ has magnitude A and unit-vector $\widehat{A}$ hence $\vec{A} = A\widehat{A}$.

(1.) (30pt) Let $\vec{A} = \langle 1, 2, 2\rangle$ and $\vec{B} = \langle 1, 0, -1\rangle$. Find the following:

(a.) $\vec{A} \bullet \vec{B}$

(b.) the angle between $\vec{A}$ and $\vec{B}$

(c.) the angle between $\vec{A}$ and the positive y-axis,

(d.) $\text{Proj}_{\vec{B}}(\vec{A})$ (the vector projection of $\vec{A}$ in the $\vec{B}$-direction)

(e.) $\vec{A} \times \vec{B}$

(2.) (10pt) Suppose $\vec{A}$ and $\vec{B}$ are orthogonal. Show that $\|\vec{A} + \vec{B}\|^2 = \|\vec{A}\|^2 + \|\vec{B}\|^2$.

(3.) (20pt) A plane $\mathcal{P}$ contains points $P = (0, 1, 5)$, $Q = (3, 3, 7)$ and $R = (-4, 0, 14)$.

(a.) (10pt) find the Cartesian equation of $\mathcal{P}$ in the form $ax + by + cz = d$

(b.) (5pt) find the area of the triangle with vertices P, Q, R

(4.) (10pt) Find the point on the plane $3x - 5y + 2z = 10$ which is closest to the point $(-9, 15, -1)$.

(5.) (10pt) Consider the path with $x = \sin t, y = e^t$ and $z = \cos t$. Find the tangential component of the acceleration of this path (we called this a_T).

(6.) (10pt) Let C be the semi-circle formed by the union of the half-circle $x^2 + y^2 = 9$ for $y \geq 0$ and the line-segment from $(-3, 0)$ to $(3, 0)$. Let $f(x, y) = xy$ and calculate $\int_C f\, ds$.

(7.) (8pt) Consider the following equations and terms:

(I.) $x^2 + y^2 + z^2/4 = 1$	**(A.)** hyberboloids of one sheet
(II.) $x^2 + y^2 = z^2$	**(B.)** hyperbolid of two sheets
(III.) $x^2 - y^2 - z^2 = 1$	**(C.)** cone
(IV.) $x^2 + y^2 - z^2 = 1$	**(D.)** ellipsoid

Fill in the blanks below with **I,II,III or IV** and **A,B,C or D** as appropriate.
Note: *patch* is another word for *parametrization*:

(a.) Equation ________ is a ________ with patch $\vec{X}(\theta, \phi) = (\cos\theta\sin\phi, \sin\theta\sin\phi, 2\cos\phi)$.

(b.) Eqn. _______ is a ______________ with patch $\vec{X}(\alpha, \beta) = (\cosh\beta, \sinh\beta\sin\alpha, \sinh\beta\cos\alpha)$

(c.) Equation ________ is a ________ with patch $\vec{X}(u, v) = (\cosh u \sin v, \cosh u \cos v, \sinh u)$.

(d.) Equation ________ is a ________ with patch $\vec{X}(\theta, t) = (t\cos\theta, t\sin\theta\sin\phi, t)$

(8.) (17pt) Suppose $\vec{\gamma}(t) = \langle 1 + 3\sin t, 2 + 4t, 3 + 3\cos t\rangle$ for $t \in \mathbb{R}$. Find the T, N, B frame for this curve. Also find the curvature and torsion for this curve.

(9.) (10pt) Suppose $\vec{\gamma}$ is the path for which T, N, B are well-defined. Recall the Frenet-Serret Equations for a path with speed v, curvature κ and torsion τ are $\frac{dT}{dt} = v\kappa N$ and $\frac{dN}{dt} = -v\kappa T + v\tau B$ and $\frac{dB}{dt} = -v\tau N$. Derive **ONE** of the following formulas:

$$\kappa = \frac{\|\gamma' \times \gamma''\|}{\|\gamma'\|^3}, \qquad \tau = \frac{(\gamma' \times \gamma'') \bullet \gamma'''}{\|\gamma' \times \gamma''\|^2}$$

(10.) (10pt) Let $\vec{b}, \vec{c}$ be nonzero unit-length perpendicular vectors and $\vec{r}_o$ is also a constant vector. Let

$$\vec{\gamma}(t) = \vec{r}_o + \cos(t)\,\vec{b} + \sin(t)\vec{c}$$

be the position of a given ninja dog on patrol. Find the dog's velocity and accleration as functions of time t. Find the curvature of the dog's patrol path. What can you tell me about this path?