MATH 497: MANIFOLD THEORY — MISSION 1 (50PTS)

Problems are typically taken from either Jeffrey Lee's text *Manifolds and Differential Geometry* (MDG) or John Lee's text *Smooth Manifolds* (SM). I've also written a few problems.

Problem 1 Spheres are a nice example. Let's build some background.

(a.) Find an atlas for the circle of radius R in $\mathbb{R}^2$.

(b.) Find an atlas for the the sphere of radius R in $\mathbb{R}^3$ and show your charts are compatible.

(c.) Find an atlas for the n-sphere of radius R in $\mathbb{R}^{n+1}$

(d.) Find a different atlas that covers most of the sphere from **(b.)** and check compatibility.

Problem 2 MDG exercise 1.40

Problem 3 MDG exercise 1.42

Problem 4 MDG exercise 1.45

Problem 5 MDG exercise 1.46

Problem 6 MDG exercise 1.47

Problem 7 MDG exercise 1.48

Problem 8 MDG exercise 1.54

Problem 9 MDG exercise 1.56

Problem 10 MDG exercise 1.60

Problem 11 MDG exercise 1.61

Problem 12 MDG Problem 4 from around page 51.

Problem 13 MDG Problem 7 from around page 51.

Problem 14 MDG Problem 16 from around page 51.

Problem 15 Let $f, g \in C^\infty(M)$. Prove the following for (U, x) a chart in the atlas of M.

(a.)

$$\frac{\partial}{\partial x^i}[f+g] = \frac{\partial f}{\partial x^i} + \frac{\partial g}{\partial x^i}$$

(b.)

$$\frac{\partial}{\partial x^i}[fg] = \frac{\partial f}{\partial x^i} g + f \frac{\partial g}{\partial x^i}.$$