Math 126, Test 3, Name:

Show your work, box answers. You are free to use a non-graphical calculator. There are 150pts to earn here plus the Bonus problem.

1. [6pts] Find constants A, B, C for which: $\frac{1}{3}\ln(x^3) - \ln(y) + 3\ln(yz) = \ln(x^A y^B z^C)$.

$$= \ln(x) - \ln(y) + \ln(y^3 z^3)$$
$$= \ln(x y^{-1} y^3 z^3)$$
$$= \ln(x^1 y^2 z^3) \hookrightarrow \boxed{\begin{array}{l} A = 1 \\ B = 2 \\ C = 3 \end{array}}$$

2. [10pts] Find the absolute extreme values for $f(x) = 3x^4 - 4x^3$ on the closed interval [0,2].

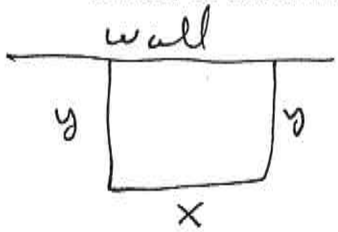
$$f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$$
$$\Rightarrow x = 0, \ x = 1 \quad \text{critical \#'s.}$$

$$f(0) = 0$$
$$f(1) = 3 - 4 = -1$$
$$f(2) = 3(2)^4 - 4(2)^3 = 48 - 32 = 16$$

$$\boxed{\begin{array}{l} \text{minimum is } -1 \text{ at } x = 1 \\ \text{max is } 16 \text{ at } x = 2 \end{array}}$$

(By Closed Interval Test)

3. [10pts] You build a chicken cage in a rectangular shape, however, one side is against a wall so you don't need to put fence on that side. If you have 100ft of fence to make the cage then what dimensions should be used in order to maximize the are enclosed by the cage?



$$x + 2y = 100$$
$$x = 100 - 2y$$

$$A = xy = (100 - 2y)y = 100y - 2y^2$$

$$\frac{dA}{dy} = 100 - 4y = 0 \quad \therefore \quad \underline{y = 25 \text{ ft}}$$

$$\Rightarrow x = 100 - 2(25) = \underline{50 \text{ ft}}$$

$$\boxed{25' \times 50'}$$

4. [32pts] Calculate dy/dx for each function given below:

a) $y = x^2 e^{6x}$

$$\frac{dy}{dx} = 2xe^{6x} + x^2 6e^{6x} = \boxed{(2x + 6x^2)e^{6x}}.$$

b) $y = \frac{e^x}{3-e^x}$

$$\frac{dy}{dx} = \frac{e^x(3-e^x) - e^x(-e^x)}{(3-e^x)^2} = \boxed{\frac{3e^x}{(3-e^x)^2}}.$$

c) $y = \ln(8x+7)$

$$\frac{dy}{dx} = \frac{1}{8x+7}\frac{d}{dx}(8x+7) = \boxed{\frac{8}{8x+7}}.$$

d) $y = (x^2+2)^x$

$$\ln y = \ln(x^2+2)^x = x\ln(x^2+2)$$

$$\frac{1}{y}\frac{dy}{dx} = \ln(x^2+2) + x\left(\frac{2x}{x^2+2}\right)$$

$$\boxed{\frac{dy}{dx} = (x^2+2)^x\left[\ln(x^2+2) + \frac{2x^2}{x^2+2}\right]}.$$

5. [10pts] If a car has velocity $v(t) = t^2 + 3$ then how far does it travel between $t = 0$ and $t = 1$?

$$\Delta x = \int_0^1 v(t)\,dt = \int_0^1 (t^2+3)\,dt = \left(\frac{t^3}{3} + 3t\right)\Big|_0^1$$

$$= \frac{1}{3} + 3 - 0$$

$$= \boxed{\frac{10}{3}}$$

6. [10pts] Suppose $f'(x) = e^x + 2x$ and $f(0) = 0$. Find $f(x)$.

antidiff. $\Rightarrow f(x) = e^x + x^2 + C$

$$f(0) = 0 = e^0 + C = 1 + C \quad \therefore \quad C = -1.$$

$$\boxed{f(x) = e^x + x^2 - 1}$$

7. [40pts] Calculate the indefinite integrals indicated below: (a, b are constants)

a) $\int (a + bx)\, dx$ $= \boxed{ax + \frac{1}{2} bx^2 + C}$

Hint: try some u-substitution for the integrals that follow.

b) $\int 33x^2(x^3+7)^{10}\, dx$

$$= \int 33x^2\, u^{10} \frac{du}{3x^2}$$
$$= \int 11 u^{10}\, du$$
$$= u^{11} + C$$
$$= \boxed{(x^3+7)^{11} + C}$$

$u = x^3 + 7$
$du = 3x^2 dx$
$dx = \frac{du}{3x^2}$

c) $\int \exp(\underbrace{2t+3}_{u})\, dt$

$du = 2dt$
$dt = du/2$

$$= \int \exp(u) \frac{du}{2}$$
$$= \frac{1}{2} \int e^u du$$
$$= \frac{1}{2} e^{2t+3} + C = \frac{1}{2} \exp(2t+3) + C$$

d) $\int \sqrt{ax+b}\, dx$

$$= \int \sqrt{u} \frac{du}{a}$$
$$= \frac{2}{3a} u^{3/2} + C$$
$$= \boxed{\frac{2}{3a}(ax+b)^{3/2} + C}$$

$u = ax + b$
$du = a dx$
$dx = du/a$ $(a \neq 0)$

OR : if $a = 0$ then $\int \sqrt{ax+b}\, dx = \int \sqrt{b}\, dx = \sqrt{b}\, x + C$.

e) $\int \frac{5x^4+2x}{x^5+x^2} dx$

$$= \int \frac{5x^4+2x}{u} \left(\frac{du}{5x^4+2x}\right)$$
$$= \int \frac{du}{u}$$
$$= \ln|u| + C$$
$$= \boxed{\ln|x^5+x^2| + C}$$

$u = x^5 + x^2$
$du = (5x^4 + 2x)dx$
$dx = \frac{du}{5x^4+2x}$

8. [32pts] Calculate the definite integrals indicated below:

a) $\int_0^1 (x+2)(x+1)\,dx$ $= \int_0^1 (x^2+3x+2)\,dx$

$= \left(\frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x\right)\Big|_0^1$

$= \frac{1}{3} + \frac{3}{2} + 2 = \frac{2+9+12}{6} = \boxed{\frac{23}{6}}$

b) $\int_{-4}^{-1} \frac{2}{x-1}\,dx$ $= 2\ln|x-1|\Big|_{-4}^{-1}$

$= 2\ln|-2| - 2\ln|-5|$

$= \boxed{2\ln(2/5)}$

$\int \frac{2dx}{x-1} = \int \frac{2\,du}{u} \leftarrow \left(\begin{array}{l} u = x-1 \\ du = dx \end{array}\right.$

$= 2\ln|u| + C$

$= 2\ln|x-1| + C$

c) $\int_5^6 (2x-9)^4\,dx$ $= \int_1^3 u^4 \frac{du}{2}$

$= \frac{u^5}{10}\Big|_1^3$

$= \boxed{\frac{1}{10}(3^5 - 1)}$

$u = 2x - 9$

$du = 2dx$

$u(5) = 10 - 9 = 1$

$u(6) = 12 - 9 = 3$

d) $\int_0^1 (e^{2x} - e^{-3x})\,dx$ $= \int_0^1 e^{2x}dx - \int_0^1 e^{-3x}dx$

$= \frac{1}{2}e^{2x}\Big|_0^1 - \left(\frac{-1}{3}e^{-3x}\right)\Big|_0^1$

$= \boxed{\frac{1}{2}(e^2 - 1) + \frac{1}{3}(e^{-3} - 1)}$

$u = ax \quad du = adx$

$\int e^{ax}dx = \int e^u \frac{du}{a}$

$= \frac{1}{a}e^u + C$

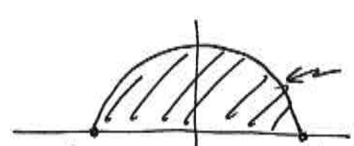
$= \frac{1}{a}e^{ax} + C$

Let $a = 2$

or $a = -3$

to get desired formulas.

Bonus[15pts]: calculate $\int_{-1}^{1} \sqrt{1-x^2}\,dx$



$y = \sqrt{1-x^2}$

-1 $\quad$ 1

$y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1$ $\quad$ CIRCLE, but $y = \sqrt{1-x^2} \rightarrow$ top half.

Hence area under $y = \sqrt{1-x^2}$ for $-1 \le x \le 1$ is $\frac{1}{2}\pi R^2$

$\rightarrow \boxed{\pi/2}$