Name (print):

Show work. Your work should be single-sided and clearly labeled. Number the pages in your solution. You will be given a folder to turn in your work, please print your full name in large font on the outside so it is easy for you to pick back up after grading. Thanks.

Recommended Practice Problems: (these are not collected)
Stewart §7.1 #'s 3, 7, 9, 11, 13, 15, 19, 21, 25, 29, 37, 41
Stewart §7.2 #'s 1, 3, 5, 7, 13, 15, 19, 21, 23, 25, 31, 33, 39, 41, 49, 55, 65, 69
Stewart §7.3 #'s 5, 7, 9 , 11, 13, 15, 17, 19, 27, 29, 31, 42
Stewart §7.4 #'s 1, 3, 5, 7, 9, 11, 13, 21, 23, 25, 27, 29, 39, 41, 43, 45, 47, 51, 53, 59, 61, 72
Stewart §7.5 #'s 1, 5, 13, 19, 21, 27, 33, 49, 57
Stewart §7.8 #'s 5, 7, 9, 11, 13, 17, 21, 25, 27, 31, 33, 37, 49, 51, 53, 57, 58, 61, 79

Problem 1 Integrals we can calculate using u-substitution and/or FTC II

(a.) $\displaystyle \int_0^1 \left(\frac{x^2 + x^3 + \sqrt{x}}{x} \right) dx$

(b.) $\displaystyle \int_{-5}^{-3} \frac{dx}{x+2}$

(c.) $\displaystyle \int \sin(3x)\, dx$

(d.) $\displaystyle \int \sec^2(2x+1)\, dx$

Problem 2 Integrals we can calculate using u-substitution and/or FTC II

(a.) $\displaystyle\int 2x \cosh(x^2)\, dx$

(b.) $\displaystyle\int_{\ln 2}^{\ln 3} \sinh x\, dx$

(c.) $\displaystyle\int \sec^3 x \tan x\, dx$

(d.) $\displaystyle\int \left(\frac{x}{x^2} + x\sqrt[3]{x} \right) dx$

Problem 3 Integrals we can calculate using u-substitution and/or FTC II

(a.) $\displaystyle \int \left(e^x \sqrt{e^x} + \frac{1}{e^x} \right) dx$

(b.) $\displaystyle \int \frac{2x\,dx}{\sqrt{1-x^4}}$

(c.) $\displaystyle \int \frac{\sin x\,dx}{1+\cos^2 x}$

(d.) $\displaystyle \int_6^7 (x-6)^{11}\,dx$

Problem 4 Calculate $\displaystyle\int x\cos(5x)\,dx$

Problem 5 Calculate $\displaystyle\int x^4 \ln x \, dx$

Problem 6 Calculate $\int \left(\sin^{-1}(x)\right)^2 dx$

Problem 7 Calculate $\int e^{-x} \cos(ax) \, dx$ where $a \neq 0$ is a constant

Problem 8 Calculate $\int \tanh^{-1}(x)\,dx$.

Problem 9 Calculate $\int_0^\pi x \sin x \cos x\,dx$

Problem 10 Show $\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$ for $n \in \mathbb{N}$. Also, use this recursive formula to calculate the integrals of $\cos^2 x$ and $\cos^4 x$.

Problem 11 Calculate $\int 2^{\sqrt{x}}\, dx$

Problem 12 Calculate $\displaystyle\int \sin^3(x)\sqrt{\cos(x)}\, dx$

Problem 13 Calculate $\displaystyle\int \sin^2(x) \sin(2x)\, dx$

Problem 14 Calculate $\displaystyle\int x \tan^2 x\, dx$

Problem 15 Calculate $\displaystyle\int_0^\pi \sin^2\theta\, d\theta$

Problem 16 Given $A \neq 0$, calculate $\displaystyle\int \sin^3(Ax)\, dx$

Problem 17 Given $A \neq 0$, calculate $\displaystyle\int \tanh(Ax)\, dx$

Problem 18 Calculate $\int_{-\pi}^{\pi} \sin(px)\sin(qx)dx$ where $p, q \in \mathbb{N}$. You should get different answers for $p = q$ and $p \neq q$.

Problem 19 Set-up, but do not determine the coefficients, of the partial fractions decomposition of:

(a.) $f(x) = \dfrac{3x^3 + 21}{(x^2 + 8x - 1)((9x^2 - 7)^2}$

(b.) $f(x) = \dfrac{2x - 42}{6x^4 + 13x^3 + 5x^2}$

Problem 20 Use long division to rewrite the given rational function as the sum of a polynomial and a proper rational function then set-up, but do not determine the coefficients, of the partial fractions decomposition for the proper rational function.

(a.) $f(x) = \dfrac{3x^4 + x^2 + 3x + 2}{x^4 - 3x^2 + 2}$

(b.) $f(x) = \dfrac{x^6 - 3x + 7}{(x^2 - 14x + 49)(x^4 - 1)}$

Problem 21 Find the partial fractions decomposition of:

(a.) $\dfrac{x^2}{(x-1)(x^2+4x+5)}$

(b.) $\dfrac{x^3+x^2+x+2}{x^4+3x^2+2}$

Problem 22 Calculate $\displaystyle\int \frac{x^2\,dx}{x^2 + 6x + 13}$

Problem 23 Calculate $\displaystyle\int \frac{2x^2 + 8}{(x-1)(x^2 + 4x + 5)}\, dx$

Problem 24 Calculate $\displaystyle\int \frac{dx}{x(x^2+1)^2}$

Problem 25 Calculate $\displaystyle\int \frac{x\,dx}{(x+1)(x+2)(x+3)}$

Problem 26 Calculate $\displaystyle\int \frac{dx}{(x-1)(x^2+1)^2}$

Problem 27 Calculate $\displaystyle\int \frac{dz}{b^2 - z^2}$ in two ways. Show how the result may either be expressed in terms of a natural logarithm or in terms of the inverse hyperbolic tangent function. Assume $b > 0$.

Problem 28 Calculate via a hyperbolic substitution, $\displaystyle\int \frac{dx}{\sqrt{x^2 + 4x + 11}}$

Problem 29 Calculate $\int (11-x^2-4x)^{-3/2}\,dx$ and use algebraic functions to state the answer as much as possible.

Problem 30 Calculate $\displaystyle\int \frac{dx}{(x^2-9)^{5/2}}$ and use algebraic functions to state the answer as much as possible.

Solution. Let $x = 3\sec\theta$, so $dx = 3\sec\theta\tan\theta\,d\theta$ and $x^2-9 = 9\tan^2\theta$, giving $(x^2-9)^{5/2} = 243\tan^5\theta$.

$$\int \frac{dx}{(x^2-9)^{5/2}} = \frac{1}{81}\int \frac{\sec\theta}{\tan^4\theta}\,d\theta = \frac{1}{81}\int \frac{\cos^3\theta}{\sin^4\theta}\,d\theta$$

With $u=\sin\theta$:

$$= \frac{1}{81}\left(\frac{1}{\sin\theta} - \frac{1}{3\sin^3\theta}\right) + C$$

Since $\sin\theta = \dfrac{\sqrt{x^2-9}}{x}$,

$$\int \frac{dx}{(x^2-9)^{5/2}} = \frac{x(2x^2-27)}{243\,(x^2-9)^{3/2}} + C$$

Problem 31 Calculate $\displaystyle\int \sqrt{1 + e^x}\, dx$

Problem 32 Calculate $\displaystyle\int \frac{\cos x \, dx}{\sin^2 x - 3\sin x + 2}$

Problem 33 Calculate $\int_0^\infty e^{-px}\,dx$ where $p > 0$.

Problem 34 Calculate $\int_0^1 \frac{dx}{\sqrt{1-x}}$.

Problem 35 Calculate $\displaystyle\int_e^\infty \frac{dx}{x\ln(x)}$.

Problem 36 Calculate $\displaystyle\int_0^1 x\ln(x)\,dx$.

Problem 37 Calculate $\displaystyle\int_1^\infty \frac{dx}{x(x+1)}$.

Problem 38 Calculate $\displaystyle\int_{-\infty}^\infty \frac{dx}{e^x + e^{-x}}$.

Problem 39 Use the comparison test for improper integrals to determine the convergence or divergence of the integrals below:

(a.) $\displaystyle\int_0^\infty \frac{dx}{(1+x^5)^{1/6}}$,

(b.) $\displaystyle\int_\pi^\infty \frac{\sin(x^2)}{x^2}\,dx$.

Problem 40 The **Laplace Transform** is important to electrical engineering and other applied Mathematics. The idea is to replace a function of time $f(t)$ with a corresponding function of frequency s denoted by $F(s)$ via the improper integration $F(s) = \int_0^\infty e^{-st} f(t)\, dt$. Calculate the Laplace transforms of the functions below:

(a.) $f(t) = e^{at}$ for $a > 0$ (you should assume $s > a$),

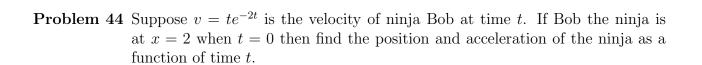(b.) $f(t) = \sin(bt)$ for $b > 0$ (assume $s > 0$).

Problem 41 Consider $f(x) = ke^{-kx}$ for $x > 0$ and $f(x) = 0$ for $x \leq 0$. Show $f(x)$ is a probability density function.

Problem 42 Let $f(x) = \begin{cases} bx & 0 \le x \le b \\ b^2 - b(x - b) & b \le x \le 2b \\ 0 & x \notin [0, 2b] \end{cases}$. For which choice of b is $f(x)$ a probability distribution ?

Problem 43 Make a $t = \tan(x/2)$ substitution to evaluate the integral:

$$\int \frac{dx}{2 + \sin(x)}.$$

An outline of the calculation: derive expressions for dx, $\sin(x)$ and $\cos(x)$ in terms of t then substitute into the given integral to reformulate the integral as a rational function.

Problem 44 Suppose $v = te^{-2t}$ is the velocity of ninja Bob at time t. If Bob the ninja is at $x = 2$ when $t = 0$ then find the position and acceleration of the ninja as a function of time t.

Problem 45 Find the area bounded by $y = \sin^2 x$ and $y = \sin^3 x$ for $0 \le x \le \pi/2$.