Name (print):

CALCULUS 132:	MISSION 2 (45PTS)

Show work. Your work should be single-sided and clearly labeled. Number the pages in your solution. You will be given a folder to turn in your work, please print your full name in large font on the outside so it is easy for you to pick back up after grading. Thanks.

Recommended Practice Problems: (these are not collected)
Stewart §9.1 #'s 1, 5, 11
Stewart §9.2 #'s 3, 4, 5, 6, 7, 9, 21
Stewart §9.3 #'s 1, 3, 5, 7, 9, 11, 15, 17, 21, 29, 31, 33, 35, 47
Stewart §9.5 #'s 5, 7, 9, 11, 15, 19, 25, 33, 35
Stewart §11.1 #'s 3, 7, 9, 11, 13, 15, 17, 23, 27, 31, 35, 39, 41, 45, 47, 49, 51, 73, 75, 77, 81
Stewart §11.2 #'s 5, 15, 17, 19, 21, 23, 25, 29, 31, 33, 37, 43, 45, 47, 53, 57, 59, 75
Stewart §11.3 #'s 3, 5, 7, 9, 11, 15, 17, 21, 25, 27, 29, 33, 34, 35
Stewart §11.4 #'s 3, 5, 7, 9, 13, 15, 23, 25, 29
Stewart §11.5 #'s 3, 5, 9, 11, 13, 17, 19, 21, 23, 25, 27, 29, 33
Stewart §11.6 #'s 7, 9, 11, 13, 15, 17, 25, 27, 29, 31, 33, 37, 39,
Stewart §11.7 #'s 1, 5, 9, 11, 15, 17, 23, 29, 33, 37

Problem 46 Use Euler's method with step-size 0.5 to compute the approximate y-values y_1, y_2, y_3 and y_4 of the solution of the initial value problem $y' = y - 2x$ with $y(1) = 0$.

Problem 47 Create a first order differential equation whose solutions have horizontal tangents at $y = -2$ and $y = 2$ and vertical tangents on the unit-circle $x^2 + y^2 = 1$. Use the pplane tool discussed in my notes to plot the direction field and a few solutions. Print that out as your solution to this problem (write the differential equation you created on the print out).

Problem 48 Solve $\dfrac{dy}{dx} = \dfrac{\cos^2 y}{1+x^2}$

Problem 49 Find the explicit solution of $\dfrac{dy}{dx} = \dfrac{x^3}{y}$ subject to the condition $y(1) = -2$.

Problem 50 Find the explicit (general) solution of $\dfrac{dy}{dx} = \dfrac{y^2 + 2y + 1}{x^2 + 3x + 2}$.

Problem 51 Find the implicit (general) solution of $\dfrac{dy}{dx} = \dfrac{\sqrt{x^2 + 4x + 5}}{\sqrt{1 - y^2}}$.

Problem 52 Find the explicit (general) solution of $\dfrac{dy}{dx} = y^4 e^x \sin(3x)$.

Problem 53 Find the implicit general solution of $\dfrac{dy}{dx} = \dfrac{\sin^{-1}(x)}{ye^y}$.

Problem 54 Find orthogonal trajectories to the curve $x^2 + y^3 = 88$.

Problem 55 Solve $\frac{dy}{dt} + 3y = e^{2t+3}$ given $y(0) = 2$.

Problem 56 Solve $y' = x - y$

Problem 57 Solve $x^2y' + 2xy = \ln x$

Problem 58 Suppose $y' = \frac{dy}{dt}$. Solve $y'' + y = 0$ using reduction of order. In particular, let $v = \frac{dy}{dt}$ and notice $y'' = \frac{dv}{dt} = \frac{dy}{dt}\frac{dv}{dy} = v\frac{dv}{dy}$.

Problem 59 Solve $y(x) = 2 + \int_1^x \frac{dt}{ty(t)}$. Notice the solution must have $y(1) = 2$.

Problem 60 Suppose the net force on an object with mass m is given by $F = -bv^2$ where b is a positive constant. Find the formula for the velocity as a function of time t where $v(0) = v_o$.

Problem 61 Calculate the limit of the sequences below:

(a.) $a_n = \dfrac{3n - 7n^2}{3 + n^2}$

(b.) $a_n = \tan^{-1}(e^{n-8} + 13)$

(c.) $a_n = 3^n 4^{-n} + 42$

(d.) $a_n = \dfrac{n + (-1)^n}{n}$

(e.) $a_n = \cos(\pi/n)$

(f.) $a_n = \cos(\pi n)$

(g.) $a_n = \ln\left(\dfrac{2n}{n+1}\right)$

Problem 62 Calculate the limit of the sequence $a_n = \dfrac{n^2(2n-1)!}{(2n+1)!}$

Problem 63 Calculate the limit of the sequence $a_n = \left(1 + \dfrac{2}{n}\right)^n$

Problem 64 Let $a_{n+1} = \left(\frac{n}{n+1}\right) a_n$ for $n \geq 1$ and suppose $a_1 = 1$. Calculate $\lim_{n\to\infty} a_n$ in the following two ways:

(a.) by applying the bounded monotonic sequence theorem as appropriate,

(b.) by calculating the explicit formula for this recursively defined sequence.

Problem 65 Prove $\lim_{n\to\infty}(r^n) = \begin{cases} 0 & |r| < 1 \\ 1 & r = 1 \\ d.n.e. & r \notin (-1, 1] \end{cases}$.

Problem 66 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{n=1}^{\infty} n \sin(1/n)$$

Problem 67 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{n=0}^{\infty} \frac{2^{3n+4}}{7^{2n+3}}$$

Problem 68 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=3}^{\infty} \big(\tan^{-1}(k) - \tan^{-1}(k+1)\big)$$

Problem 69 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{n=1}^{\infty} \frac{k^2}{(2k+1)!}$$

Problem 70 Show that $\dfrac{2^n}{n!} \leq \dfrac{4}{n}$ and use this to calculate $\lim\limits_{n\to\infty} \dfrac{2^n}{n!}$.

Problem 71 Consider the finite sum $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{17}$. Express this sum in two ways:

(a.) as a sum over k which begins at $k = 0$,

(b.) as a sum over n which begins at $n = 3$.

Problem 72 Calculate $\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+3)}$.

Problem 73 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=1}^{\infty} \frac{\tan^{-1}(k)}{k^2+1}$$

Problem 74 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)}$$

Problem 75 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=1}^{\infty} \frac{1+2^k}{1+5^k}$$

Problem 76 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=1}^{\infty} \frac{2+\cos(k)}{\sqrt{k+1}}$$

Problem 77 Determine what n is needed for the error in $S_n = \sum_{k=1}^{n} \frac{1}{k^3}$ to be at most 0.0001 with respect to the $p = 3$ series.

Problem 78 Determine what n is needed for the error in $S_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k^3}$ to be at most 0.0001 with respect to the alternating $p = 3$ series.

Problem 79 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=1}^{\infty} \frac{2^k k!}{k^k}$$

Problem 80 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=1}^{\infty} \left(\frac{k}{2k+1} \right)^k$$

Problem 81 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=1}^{\infty} \frac{2k + \sqrt{k}}{k^3 + \sqrt{k}}$$

Problem 82 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

Problem 83 Carefully explain if the following series converges or diverges by the appropriate application of theory we have discussed.

$$\sum_{k=1}^{\infty} \frac{k^k}{(3^k)^2}$$

Problem 84 Show $\sum_{k=3}^{\infty}(-1)^k \frac{\ln k}{k}$ is conditionally convergent.

Problem 85 Show $\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k}$ is absolutely converent.

Problem 86 Suppose A, B are real numbers and a_n, b_n are real values for each n. Prove the following:

(a.) If $a_n \to A$ and $b_n \to B$ as $n \to \infty$ then $a_n + b_n \to A + B$

(b.) If $c \in \mathbb{R}$ then $\lim_{n\to\infty}(ca_n) = cA$.

Problem 87 Define what it means for a series $\sum_{n=1}^{\infty} a_n$ to sum to L where L is a real number and a_n denotes the n-th term in the series of real numbers.

Problem 88 Suppose $S_n = \displaystyle\sum_{k=1}^{n} a_k$ where $a_k \in \mathbb{R}$ for each $k \in \mathbb{N}$. Given $\displaystyle\sum_{n=1}^{\infty} a_n = S$ for some $S \in \mathbb{R}$ prove that $\lim_{n\to\infty} a_n = 0$.

Problem 89 Suppose $\displaystyle\sum_{n=1}^{\infty} a_n = A$ and $\displaystyle\sum_{n=1}^{\infty} b_n = B$ where $a_n, b_n \in \mathbb{R}$ for each $n \in \mathbb{N}$ and A, B are real values. If $c \in \mathbb{R}$ then prove $\displaystyle\sum_{n=1}^{\infty}(ca_n + b_n) = cA + B$.

Problem 90 Use FTC II to find the limit of the sequence below:

$$a_n = \sum_{k=1}^{n} \left(1 + \frac{3k}{n}\right)^4 \frac{3}{n}$$