

Name (print):

CALCULUS 132:

MISSION 4 (65PTS)

Show work. Your work should be single-sided and clearly labeled. Number the pages in your solution. You will be given a folder to turn in your work, please print your full name in large font on the outside so it is easy for you to pick back up after grading. Thanks.

**Recommended Practice Problems:** (these are not collected)

Stewart §10.1 #'s 5, 7, 8, 9, 10, 17, 21, 28, 29, 33, 34, 35, 37, 51

Stewart §10.2 #'s 1, 3, 5, 7, 9, 11, 17, 31, 33, 37, 39, 41, 43, 45, 59, 71, 74

Stewart §10.3 #'s 1, 3, 5, 7, 9, 15, 17, 21, 23, 33, 35, 37, 41, 45, 54, 55, 59, 63, 67, 69, 71

Stewart §10.4 #'s 1, 3, 5, 7, 9, 11, 19, 21, 23, 29, 31, 33, 35, 39, 41, 47, 49

Stewart §10.5 #'s 3, 5, 7, 11, 17, 18, 19, 20, 23, 25, 27, 29, 33, 39, 43

Stewart §10.6 #'s 9, 11, 19, 23, 28, 30

**Problem 126** For each pair of points  $P, Q$  find the vector  $\overrightarrow{PQ} = Q - P$  and calculate  $\overrightarrow{PQ} \cdot \overrightarrow{PQ}$  and find the distance  $d(P, Q) = \sqrt{\overrightarrow{PQ} \cdot \overrightarrow{PQ}}$  between the given points.

(a.)  $P = (1, 3)$  and  $Q = (8, 8)$

(b.)  $P = (-3, 1)$  and  $Q = (5, -2)$

**Problem 127** For the given pair  $\vec{A}, \vec{B}$  calculate  $\vec{A} \cdot \vec{B}$  and find the  $\angle(\vec{A}, \vec{B})$ .

(a.)  $\vec{A} = \langle 1, 3 \rangle$  and  $\vec{B} = \langle -1, 2 \rangle$

(b.)  $\vec{A} = \langle 1, 2 \rangle$  and  $\vec{B} = \langle 2, -1 \rangle$

(c.)  $\vec{A} = \langle 1, 1 \rangle$  and  $\vec{B} = \langle 2, 0 \rangle$

**Problem 128** For each  $\vec{A}$ , find its unit-vector  $\hat{A}$ , standard angle, and magnitude  $A$ .

(a.)  $\vec{A} = \langle -1, -3 \rangle$

(b.)  $\vec{A} = \langle 2, 4 \rangle$

(c.)  $\vec{A} = \langle -3, 4 \rangle$

**Problem 129** Let  $\vec{R}(t) = \langle t^2, t^3 \rangle$  for  $t \in \mathbb{R}$ .

(a.) Calculate  $\frac{d\vec{R}}{dt}$

(b.) Calculate  $\frac{d\vec{R}}{dt} \cdot \vec{R}$

**Problem 130** Let  $\vec{R}(t) = \langle \cos(t^2), \sin(t^2) \rangle$  for  $t \in \mathbb{R}$ .

(a.) Calculate  $\frac{d\vec{R}}{dt}$

(b.) Calculate  $\frac{d\vec{R}}{dt} \cdot \vec{R}$

**Problem 131** Stewart §10.1 # 11

**Problem 132** Stewart §10.1 # 15

**Problem 133** Stewart §10.1 # 19

**Problem 134** Stewart §10.1 # 45 (use technology to graph)

**Problem 135** Stewart §10.1 # 46c

**Problem 136** Consider a path parametrized by  $x = 1/t$  and  $y = t^2 + 1$  and find the

(a.) Find the Cartesian equation for the path

(b.) Cartesian equation of the tangent line to the path at  $t = 1$ .

(c.) the parametrization of the colliding tangent line at  $t = 1$



**Problem 137** Consider a line-segment  $L$  which goes from  $(3, 5)$  to  $(-2, 13)$ .

(a.) Find the parametric equations for  $L$  by using a parameter  $t$  which ranges over  $0 \leq t \leq 1$ .

(b.) Find a vector-valued function of time  $t \in [0, 1]$  which parametrizes  $L$ ; provide a formula for  $\vec{r}: [0, 1] \rightarrow L$  serves as a parametrization of  $L$ .

**Problem 138** Find a parametrization of the piecewise linear path from  $P = (0, 0)$  to  $Q = (1, 2)$  to  $R = (4, -3)$ . Create formulas which put  $P$  at  $t = 0$ , then  $Q$  at  $t = 1/2$  then  $R$  at  $t = 1$ .

**Problem 139** In each case find a vector parametrization of the given curve. That is, provide a formula for  $\vec{r}: I \subseteq \mathbb{R} \rightarrow C$  serves as a parametrization of  $C$ .

(a.)  $C$  is given by  $x^2 - 2x + y^2 - 6y = 6$  which has  $x \geq 1$ .

(b.)  $x^2 - 4y^2 = 1$  which has  $x > 0$  and  $y \geq 0$ .

(c.)  $x - y = 1$  bounded by  $x^2 + y^2 = 4$

**Problem 140** Let  $\vec{v}(t) = \langle \sin(t), t^3 \rangle$  denote the velocity of some particle at time  $t$ . You are given that the particle is at  $(2, 1)$  at time zero.

(a) the acceleration as a function of time  $t$

(b) the position as a function of time  $t$

**Problem 141** Stewart §10.2 # 4

**Problem 142** Stewart §10.2 # 8

**Problem 143** Stewart §10.2 # 10

**Problem 144** Stewart §10.2 # 13

**Problem 145** Consider the curve  $C$  with parametrization  $\vec{r}(t) = \langle 2 + 3 \sin t, 2 \cos t \rangle$  for  $0 \leq t \leq 2\pi$ . Calculate  $\frac{d^2y}{dx^2}$  as a function of time  $t$  for the given curve and analyze where  $C$  is concave up or down. Sketch the curve and check that your calculus gives reasonable results.

**Problem 146** Stewart §10.2 # 18

**Problem 147** Stewart §10.2 # 29

**Problem 148** Stewart §10.2 # 34

**Problem 149** Stewart §10.2 # 42

**Problem 150** Stewart §10.2 # 44

**Problem 151** Stewart §10.2 # 48



**Problem 152** Stewart §10.2 # 51

**Problem 153** Stewart §10.2 # 57

**Problem 154** Suppose  $x = e^t$  and  $y = \sin(t)$  for  $0 \leq t \leq \pi$

- (a.) all horizontal tangents
- (b.) all vertical tangents
- (c.) an integral representation for the arclength of  $C$ . Use technology to calculate the integral.
- (d.) an integral representation for surface area of of the surface obtained by rotating the given path around the  $x$ -axis. Use technology to calculate the integral.





**Problem 157** Stewart §10.3 # 2

**Problem 158** Stewart §10.3 # 4

**Problem 159** Stewart §10.3 # 6

**Problem 160** Stewart §10.3 # 11

**Problem 161** Stewart §10.3 # 16

**Problem 162** Stewart §10.3 # 33

**Problem 163** Stewart §10.3 # 35

**Problem 164** Stewart §10.3 # 43



**Problem 165** Stewart §10.3 # 57

**Problem 166** Stewart §10.3 # 61

**Problem 167** Stewart §10.3 # 65

**Problem 168** Find the equation of the tangent line to  $r^2 - 3r + 2 = 0$  at the point  $(2, 0)$ .

**Problem 169** Stewart §10.4 # 6

**Problem 170** Stewart §10.4 # 13

**Problem 171** Stewart §10.4 # 17

**Problem 172** Consider the polar curve  $r = 1 + \sin(\theta)$ . Find:

- (a) a plot of this curve
- (b) the area enclosed by the curve
- (c) an integral which represents the arclength of the curve  
(DO NOT ATTEMPT this integral!)

**Problem 173** Consider the polar curve  $r = \sin(3\theta)$ . Find:

- (a) a plot of this curve
- (b) the area enclosed by the curve
- (c) an integral which represents the arclength of the curve  
(DO NOT ATTEMPT this integral!)

**Problem 174** Consider the polar curve  $r = \theta^2$  for  $0 \leq \theta \leq \pi$ . Find:

- (a) a plot of this curve
- (b) the area enclosed by the curve and the  $x$ -axis
- (c) the arclength of the curve.

**Problem 175** Stewart §10.4 # 27



**Problem 176** Consider the area bounded outside  $r = 1$  and inside  $r = 2 \cos(2\theta)$ . Sketch these curves and find the area.

**Problem 177** Calculate the area of the intersection of the areas bounded by  $r = 2 \sin(\theta)$ ,  $r = 2 \cos(\theta)$  and  $r = 1$ .

**Problem 178** Find the slope of the curve with parametric polar equations  $r = t^2$  and  $\theta = e^t$  at the point where  $t = \ln(2)$ .

**Problem 179** Stewart §10.4 # 37

**Problem 180** Stewart §10.4 # 45

**Problem 181** Stewart §10.4 # 51

**Problem 182** If  $\vec{A}$  and  $\vec{B}$  are functions from  $\mathbb{R}$  to  $\mathbb{R}^2$  which are both differentiable functions of time  $t$  then we should prove in lecture that

$$\frac{d}{dt}(\vec{A} \cdot \vec{B}) = \frac{d\vec{A}}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\vec{B}}{dt}$$

The equation of a particle moving on a circle at position  $\vec{r}$  which has center  $\vec{c}$  and radius  $R$  is given by  $\|\vec{r} - \vec{c}\| = R$ . Show that a particle moving in a circle as above has:

- (a.) velocity  $\vec{v} = \frac{d\vec{r}}{dt}$  is perpendicular to  $\vec{r} - \vec{c}$  at each time  $t$ ,
- (b.) and if  $v$  is constant then show  $a = v^2/R$  and  $\hat{a}$  is in the direction of  $\vec{c} - \vec{r}$ .

**Problem 183** If  $v$  denotes speed of a particle then  $v^2 = \dot{x}^2 + \dot{y}^2$  where  $\dot{x} = \frac{dx}{dt}$  and  $\dot{y} = \frac{dy}{dt}$ . If  $x = r \cos \theta$  and  $y = r \sin \theta$  then show  $v^2 = \dot{r}^2 + r^2 \dot{\theta}^2$ .

**Problem 184** Stewart §10.5 # 15

**Problem 185** Stewart §10.5 # 20

**Problem 186** Stewart §10.5 # 61

**Problem 187** Stewart §10.6 # 10

**Problem 188** Stewart §10.6 # 12



**Problem 189** Stewart §10.6 # 25

**Problem 190** Stewart §10.6 # 29