NAME:

CALCULUS 132:	QUIZ 5: FALL 2016

Show work for credit. Thanks. You are allowed the use of a calculator. No electronic communication devices of any kind permitted. You are allowed your notes and your own written solutions to the homework (not print-outs of my solution). Box answers.

Problem 1 (2pts) Determine if $y_1 = e^{-x}$ or $y_2 = \sin(2x)$ is a solution to $y'' + 4y = 0$. Why or why not? Explain which is a solution and which is not.

$y_1' = -e^{-x}$ and $y_1'' = e^{-x}$ hence $y_1'' + 4y_1 = e^{-x} + 4e^{-x} = 5e^{-x} \neq 0$.

$y_2' = 2\cos(2x)$ and $y_2'' = -4\sin(2x) = -4y_2''$ $\therefore$ $y_2'' + 4y_2 = 0$

Hence y_2 is sol^= whereas y_1 is not a sol^=

Problem 2 (5pts) Solve **one** of the following:

(a) Find the general solution of $\dfrac{dy}{dx} = \dfrac{\cos(y^2)}{y\sqrt{x}}$.

(b) Solve $\frac{dy}{dx} = x\sqrt{\frac{1-y^2}{1-x^2}}$ given that $y(0) = 0$.

(a.) $\dfrac{dy}{dx} = \dfrac{\cos(y^2)}{y\sqrt{x}} \Rightarrow \dfrac{y\,dy}{\cos(y^2)} = \dfrac{dx}{\sqrt{x}} \Rightarrow \int y\sec(y^2)dy = \int \dfrac{dx}{\sqrt{x}}$

$\frac{1}{2}\int \sec(w)\,dw = 2\sqrt{x} + C$

$\frac{1}{2}\ln|\sec w + \tan w| = 2\sqrt{x} + C$

$\boxed{\frac{1}{2}\ln|\sec(y^2) + \tan(y^2)| = 2\sqrt{x} + C}$

(b.) $\dfrac{dy}{dx} = x\sqrt{\dfrac{1-y^2}{1-x^2}} \Rightarrow \int \dfrac{dy}{\sqrt{1-y^2}} = \int \dfrac{x\,dx}{\sqrt{1-x^2}}$

$\sin^{-1}(y) = \frac{1}{2}\int \frac{dw}{\sqrt{w}} = 2\sqrt{w} + C = 2\sqrt{1-x^2} + C$

$y(0) = 0 \Rightarrow 0 = 2 + C \therefore \boxed{\sin^{-1}(y) = 2\sqrt{1-x^2} - 2}$

$\boxed{\text{or } y = \sin[2(\sqrt{1-x^2} - 1)]}$

Problem 3 (5pts) Find the orthogonal trajectories to the family of curves with $x^2y^3 = C$.

$x^2y^3 = C \Rightarrow 2x + 3x^2y^2\dfrac{dy}{dx} = 0$

$\therefore \dfrac{dy}{dx} = \dfrac{-2x}{3x^2y^2} \quad \therefore \left.\dfrac{dy}{dx}\right|_{OT} = \dfrac{3x^2y^2}{2x}$

Solve $\dfrac{dy}{dx} = \dfrac{3x^2y^2}{2x} \Rightarrow \int \dfrac{dy}{y^2} = \dfrac{3x\,dx}{2} = \int \dfrac{3x\,dx}{2}$

$\therefore \boxed{\dfrac{-1}{y} = \dfrac{3}{4}x^2 + C}$ aka $\boxed{y = \dfrac{-1}{\frac{3}{4}x^2 + C}}$

Problem 4 (5pts) Solve $\frac{dy}{dx} - \frac{2}{x}y = \sin(1/x)$

$$I = \exp\left(\int \frac{-2\,dx}{x}\right) = \exp\left(-2\ln|x|\right) = \exp\left(\ln|x|^{-2}\right) = |x|^{-2} = \frac{1}{x^2}$$

Thus, multiplying by $\frac{1}{x^2}$ yields,

$$\frac{1}{x^2}\frac{dy}{dx} - \frac{2}{x^3}y = \frac{1}{x^2}\sin\left(\frac{1}{x}\right)$$

$$\frac{d}{dx}\left[\frac{1}{x^2}y\right] = \frac{1}{x^2}\sin\left(\frac{1}{x}\right) \qquad u = \frac{1}{x}$$

$$du = \frac{-1}{x^2}dx$$

$$\Rightarrow \frac{1}{x^2}y = \int \frac{1}{x^2}\sin\left(\frac{1}{x}\right)dx = -\int \sin(u)\,du = \cos(u) + C$$

$$\therefore \boxed{y = x^2\cos\left(\frac{1}{x}\right) + Cx^2}$$

aka $\underline{\frac{y}{x^2} = \cos\left(\frac{1}{x}\right) + C}$

Problem 5 (5pts) Suppose a unit-mass ($m = 1$) has a force of $F = te^{t+v}$ applied. Find the velocity as a function of time.

$$F = m\frac{dv}{dt} = te^{t+v} \Rightarrow \frac{dv}{dt} = te^te^v$$

$$\int e^{-v}dv = \int te^t\,dt$$

$$-e^{-v} = te^t - e^t + \bar{C}$$

$$\therefore e^{-v} = c - te^t + e^t$$

$$-v = \ln(c - te^t + e^t)$$

$$\boxed{v = \ln\left(\frac{1}{c - te^t + e^t}\right)}$$

aka $\underline{v(t) = -\ln(c - te^t + e^t)}$

Remark:

$$\int \underbrace{t}_{u}\underbrace{e^t dt}_{dv} = uv - \int v\,du$$

$$= te^t - \int e^t dt$$

$$= te^t - e^t + c$$

in case we forgot