Physics 231: Test 1: Name:

Show your work. **Box your answers.** No graphing calculators or other electronic communication devices allowed. There are at least 200pts to earn here. Answers must be given proper units and vector notation where appropriate. Thanks and enjoy! Assume $g = 9.8\ m/s^2$ throughout this test.

[Problem 1][10pts] If a skydiver with mass $M = 70\ kg$ has a drag-force of $F_d = bv^2$ and a terminal velocity of $v = 30\ m/s$ then **what is the value of drag-coefficient b?**

$$a = 0 \Rightarrow mg = bv^2 \Rightarrow b = \frac{mg}{v^2} = \frac{(70kg)(9.8\frac{m}{s^2})}{(30m/s)^2} = \boxed{0.762\ \frac{kg}{m}}$$

[Problem 2][20pts] Suppose that $v(t) = At^2 + Bsin(Ct)$ gives the velocity of a mass M undergoing one-dimensional motion. If the mass is at the origin when $t = 0$ then find

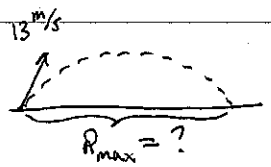
a. **Acceleration at time t**

$$a = \frac{dv}{dt} = \boxed{2At + BC\cos(Ct) = a(t)}$$

b. **Position at time t**

$$x(t) = x(0) + \int_0^t [A\tau^2 + B\sin(C\tau)]\,d\tau$$
$$= 0 + \frac{1}{3}At^3 - \frac{B}{C}(\cos(t) - 1)$$
$$\boxed{x(t) = \frac{1}{3}At^3 - \frac{B}{C}(\cos t - 1)}$$

[Problem 3] [10pts] A toy gun has a muzzle speed of 13m/s. What is its **maximum range** on level ground?



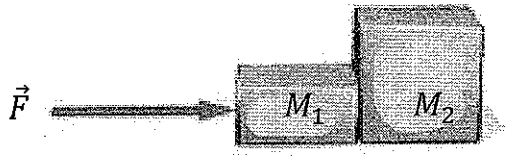
$$R(\theta) = \frac{V_0^2 \sin(2\theta)}{g} \quad \text{has } R_{max} \text{ for } \theta = 45°$$

$$R_{max} = \frac{V_0^2}{g} = \frac{(13\ m/s)^2}{9.8\ m/s^2} = \boxed{17.24m}$$

[Problem 4][10pts] A remote control car with mass 1.2 kg travels in a circle with radius 2.00 m at a constant speed of 2.00 m/s. What is the **force of friction** needed to maintain this motion?

$$F_f = \underbrace{\mu mg = \frac{mv^2}{R}}_{\text{circular motion}} = \frac{(1.2kg)(2.00\ m/s)^2}{2.00m} = \boxed{2.4N}$$

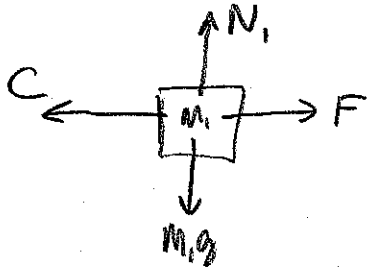
[Problem 5][30pts] Two ice blocks with mass M_1 and M_2 slide across a frictionless surface as they pushed by a horizontal force $\vec{F}$ with a magnitude $(M_1 + M_2)g$. What is the **acceleration** of the system and what is the **contact force** between the masses?

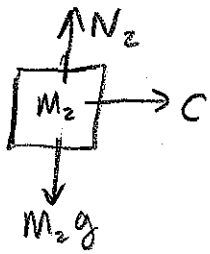


$$F = (M_1 + M_2)a$$

$$\therefore a = \frac{(M_1+M_2)g}{M_1+M_2} = g$$

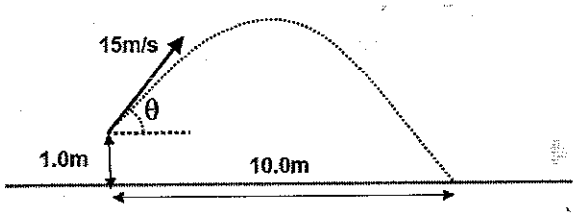
acceleration!





$M_1 a = F - C$ $\quad$ $M_2 a = C$ $\quad \therefore$ $\boxed{C = m_2 g}$

[Problem 6] [25pts] You strike a baseball with a bat 1.00m above the ground such that the ball leaves the bat at 15m/s. If the ball travels 10m horizontally before striking the ground then **at what θ was the ball hit?**



$$y(t) = 1 + (15\sin\theta)t - 4.9t^2 \quad , \quad X(t) = (15\cos\theta)t$$

$$10 = 15\cos\theta\, t \quad \therefore \quad t = \frac{10}{15\cos\theta}$$

$$0 = 1 + (15\sin\theta)t - 4.9t^2$$

$$0 = 1 + 15\sin\theta\left(\frac{10}{15\cos\theta}\right) - 4.9\left(\frac{10}{15\cos\theta}\right)^2$$

$$0 = 1 + 10\tan\theta - 4.9\left(\frac{100}{225}\right)\sec^2\theta \qquad \tan^2\theta + 1 = \sec^2\theta$$

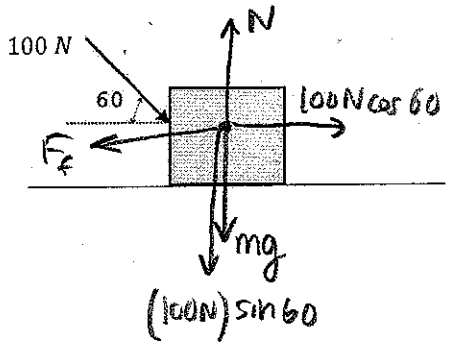
$$0 = 1 + 10z - 2.178(1 + z^2)$$

$$0 = -2.178z^2 + 10z - 1.178$$

$$z = \frac{-10 \pm \sqrt{100 - 4(2.178)(1.178)}}{2(-2.178)} = \frac{-10 \pm 9.473}{-4.356} = 0.121$$

$$\theta = \tan^{-1}(0.121) = \boxed{6.9^\circ}$$

[Problem 7][25pts] Suppose you push a box with a force $\vec{F}$ with $F = 100\ N$ directed 60 degrees above the horizontal. If the box accelerates at $2\ m/s^2$ across a plane with coefficient of kinetic friction 0.5 then **what is the mass of the box?**



$$ma = 100\cos 60° - 0.5N$$
$$0 = N - mg - 100\sin 60°$$ } in Newtons.

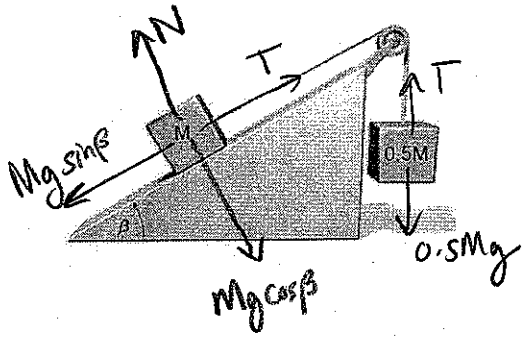
$$2ma = 100 - N$$
$$0 = N - mg - 86.6$$

$$2ma = 100 - mg - 86.6$$

$$m = \frac{13.4\ N}{2(2\ m/s^2) + 9.8\ m/s^2}$$

$$\boxed{m = 0.971\ kg}$$

[Problem 8] [30pts] Suppose an inclined plane has a coefficient of static friction 0.8 and a coefficient of kinetic friction of 0.2. If the mass $M_1 = M$ is connected to a mass $M_2 = 0.5M$ by a massless rope over the frictionless pulley as pictured below then:



a. **Find the acceleration of M if $\beta = 10°$**

b. **Find the acceleration of M if $\beta = 50°$**

depends on sign of $T - Mg\sin\beta$

$$N = Mg\cos\beta \Rightarrow ma = T - Mg\sin\beta \pm F_f$$

a.) Equilibrium? $a = 0 \Rightarrow T = 0.5Mg$

$$\Rightarrow 0 = 0.5Mg - Mg\sin\beta - F_f$$
$$\Rightarrow (0.5 - \sin\beta)Mg = F_f \leq 0.8Mg\cos\beta$$
$$\Rightarrow 0.5 - \sin\beta \overset{(?)}{\leq} 0.8\cos\beta$$

$0.326 \overset{?}{\leq} 0.788$ $(\beta = 10°)$

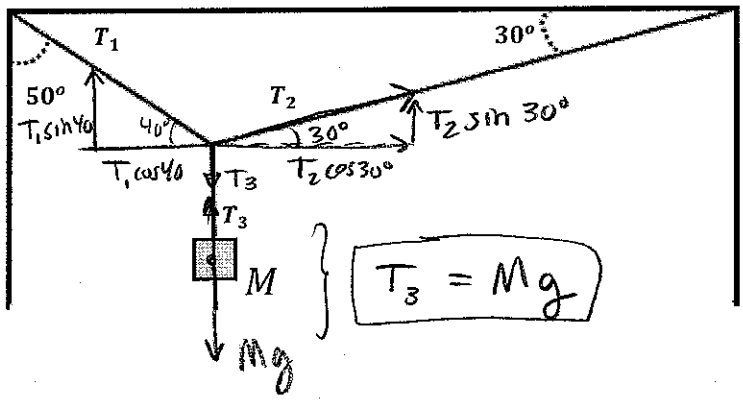
$-0.266 \overset{?}{\leq} 0.514$ $(\beta = 50°)$

(net, non-friction force goes down plane)

In both cases F_f is sufficient to give $\boxed{a = 0}$

(in $\beta = 50°$ case F_f points up the plane.)

[Problem 9] [30pts] Find the tensions T_1, T_2, T_3 in each rope as labeled below in terms of the given mass M and and gravitational acceleration g. Assume the ropes cannot stretch and the system is in equilibrium.



$T_3 = Mg$

$T_2 \cos 30° - T_1 \cos 40° = 0 \Rightarrow T_2 = T_1 \left(\frac{\cos 40°}{\cos 30°}\right)$

$-Mg + T_1 \sin 40° + T_2 \sin 30° = 0$

$-Mg + T_1 \sin 40° + T_1 \left(\frac{\cos 40°}{\cos 30°}\right) \sin 30° = 0$

$T_1 = \frac{Mg}{\sin 40 + \cos 40 \tan 30} = 0.922Mg$

$T_2 = 0.815Mg$

(work ONE of the following, thanks!)

[Problem 10a] [20pts] Suppose projectiles are launched at angle θ off a building of height H at a speed v_o. Show that the speed v_f with which the projectile hits the ground is independent of the launch angle θ.

[Problem 10b] [20pts] Two stones are dropped from a cliff. The second stone is droped 1.6 seconds after the first. **How far below the top of the cliff is the second stone** when the distance between the stones is 36 meters?

See Notes. LECTURE 4 [E14]. Recall I mentioned in lecture this was an interesting example worth studying.

LECTURE 6 [E5] & [E6]

Soon we learn energy methods to make this easy...