Name: ______________________________

PHYSICS 231, SPRING 2023. TEST I (150PTS+20PTS)

Your solutions should be neat, correct and complete. Full credit is not necessarily attained from the correct answer, you can lose points if the solution is not readable. **Box answer** where appropriate. You may omit units on calculations, however, answers must include proper units for full-credit. You are allowed a 3" × 5" card. By default, problems based on earth with $g = 9.8m/s^2$.

**Problem 1** (15pt) Consider a mass $M$ undergoing one-dimensional motion in the $x$-direction. At time $t = 0$ the mass is at position $x = x_o$. The velocity at time $t$ is given by $v = \alpha + \beta t^3$ where $\alpha, \beta$ are constants. (your answers will involve the given constants)

(a) Find the net-force on $M$ at time $t$

$$a = \frac{dV}{dt} = \frac{d}{dt}\left(\alpha + \beta t^3\right) = 3\beta t^2 \Rightarrow F_{net} = ma = \boxed{3m\beta t^2}$$

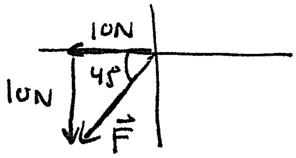
(b) Find the position of $M$ at time $t$.

$$\frac{dx}{dt} = V = \alpha + \beta t^3 \Rightarrow X = \alpha t + \frac{1}{4}\beta t^4 + C$$

$$X(0) = X_0 = 0 + 0 + C \quad \therefore \quad \boxed{X = X_0 + \alpha t + \frac{1}{4}\beta t^4}$$

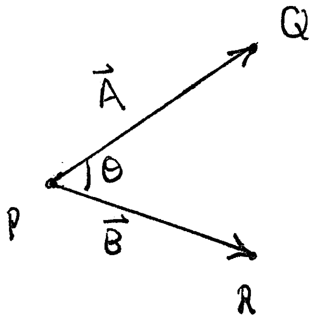
**Problem 2** (10pts) Find the magnitude and standard angle for $\vec{F} = -(10\,N)\,\hat{x} - (10\,N)\,\hat{y}$.

$$F = \sqrt{(10N)^2 + (10N)^2} = \sqrt{200}\ N = \boxed{14.14N}$$



$$\boxed{\theta = 225°}$$

**Problem 3** (10pts) Suppose $P = (0,1,1)$ and $Q = (-3,2,8)$ and $R = (2,0,2)$. Find the angle at $P$ interior to the triangle $PQR$.



$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

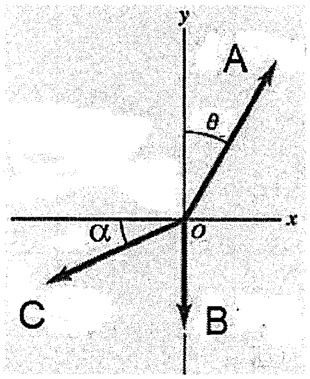
$$\vec{A} = \overrightarrow{PQ} = Q - P = \langle -3, 1, 7 \rangle, \quad A = \sqrt{9+1+49} \doteq 7.68$$

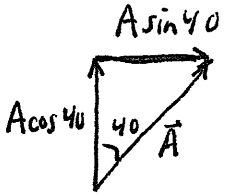
$$\vec{B} = \overrightarrow{PR} = R - P = \langle 2, -1, 1 \rangle, \quad B = \sqrt{4+1+1} \doteq 2.45$$

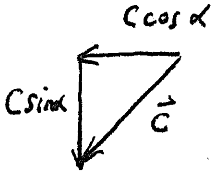
$$\vec{A} \cdot \vec{B} = -3(2) + 1(-1) + 7 = 0 = AB\cos\theta$$

$$\cos\theta = 0 \longrightarrow \theta = \cos^{-1}(0) = \boxed{90°}$$

**Problem 4** (15pt) Let the vectors $\vec{A}, \vec{B}, \vec{C}$ pictured below have magnitudes $A = 10$, $B = 5$, $C = 8$ where $\theta = 40^o$ and $\alpha = 30^o$. Find the magnitude and standard angle of $\vec{D} = \vec{A} - \vec{B} + \vec{C}$.







$A = 10$
$B = 5$
$C = 8$

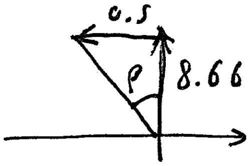
$\vec{A} = \langle A\sin 40°, A\cos 40° \rangle = \langle 6.428, 7.660 \rangle$

$\vec{B} = \langle 0, -5 \rangle$

$\vec{C} = \langle -C\cos 30°, -C\sin 30° \rangle = \langle -6.928, -4 \rangle$

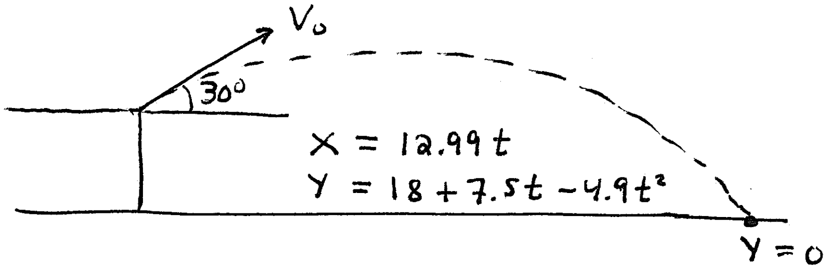
$\vec{D} = \vec{A} - \vec{B} + \vec{C} = \langle 6.428 - 0 - 6.928, 7.660 + 5 - 4 \rangle = \langle -0.5, 8.66 \rangle$

$D = \sqrt{(-0.5)^2 + (8.66)^2} = \boxed{8.674}$ ← magnitude



$\rho = \tan^{-1}\left(\frac{0.5}{8.66}\right) = 3.3° \Rightarrow \boxed{\angle \vec{D} = 93.3°}$

**Problem 5** (15pt) A man stands on the roof of a 18.0 $m$ tall building and throws a rock with a velocity of magnitude 15.0 $m/s$ at an angle of 30.0$^o$ above the horizontal. You can ignore air resistance. Assuming the building is on a horizontal plane, **find how far the rock travels horizontally from the point where it was thrown.**



$V_{0x} = (15 \frac{m}{s})\cos 30° = 12.99 \, m/s$

$V_{0y} = (15 \frac{m}{s})\sin 30° = 7.5 \, m/s$

Solve $-4.9t^2 + 7.5t + 18 = 0$

$$t = \frac{-7.5 \pm \sqrt{(7.5)^2 - 4(-4.9)(18)}}{-9.8} = \underbrace{2.83}_{} \text{ or } -1.298$$

physically interesting solution

$\Delta x \cong (12.99 \, m/s)(2.83 s) = \boxed{36.76 m}$

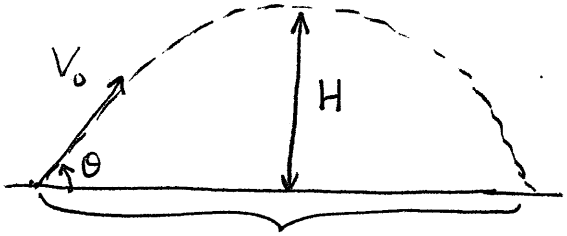
**Problem 6** (10pts) Given the position on mass $m$ at time $t$ is $\vec{r} = \langle \alpha t, \beta t^2, \gamma t^3 \rangle$ find the speed and net-force on $m$:

$$\vec{V} = \frac{d\vec{r}}{dt} = \langle \alpha, 2\beta t, 3\gamma t^2 \rangle \Rightarrow \boxed{V = \sqrt{\alpha^2 + 4\beta^2 t^2 + 9\gamma^2 t^4}}$$

(speed is the magnitude of the velocity)

$$\vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} \Rightarrow \boxed{\vec{F} = \langle 0, 2\beta m, 6\gamma m t \rangle}$$

**Problem 7** (15pts) Pastor Chuck throws a roll in such a way that its horizontal range is equal to five times its maximum height. At what angle of inclination was roll thrown ? Assume a level landscape and ignore air friction (treat the roll as a projectile).



$$R = 5H$$

$$R = \frac{V_0^2 \sin(2\theta)}{g} = \frac{2V_0^2 \sin\theta\cos\theta}{g} \quad \& \quad H = \frac{V_0^2 \sin^2\theta}{2g}$$

$$R = \frac{2V_0^2 \sin\theta\cos\theta}{g} = 5\left(\frac{V_0^2 \sin^2\theta}{2g}\right)$$

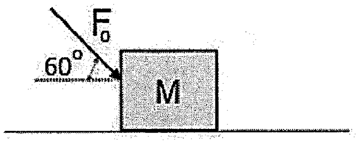
Cancel $V_0^2$, $g$ and $\sin\theta$ to yield,

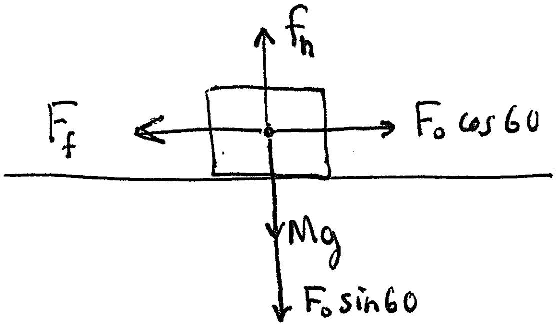
$$2\cos\theta = \frac{5\sin\theta}{2}$$

$$\tan\theta = \frac{4}{5}$$

$$\theta = \tan^{-1}\left(\frac{4}{5}\right) = \boxed{38.66^\circ}$$

**Problem 8** (10pts) Suppose the coefficient of ~~static~~ kinetic friction is $\mu_k = 0.6$ for the box sliding along the plane pictured below. If $F_o = 2Mg$ then find the acceleration of $M$





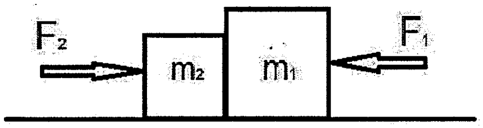
$$f_n = Mg + F_o \sin 60$$

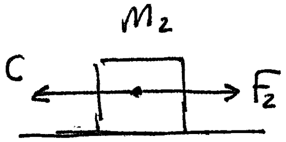
$$f_n = Mg + 2Mg \sin 60$$

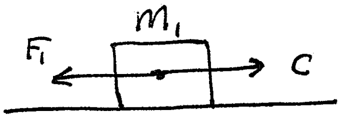
$$f_n = 2.732Mg \Rightarrow F_f = 0.6 f_n = 1.639Mg$$

$$\begin{aligned} ma &= F_o \cos 60 - F_f \\ &= 2Mg(0.5) - 1.639Mg \\ &= -0.639Mg \longrightarrow \boxed{a = -0.639g = -6.264\,m/s^2} \end{aligned}$$

**Problem 9** (15pts) Suppose forces $F_1$ and $F_2$ push on masses $m_1$ and $m_2$ on a frictionless plane. Find the contact force between the masses.







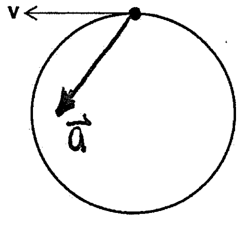
$$\begin{aligned} &\begin{pmatrix} m_1 a = C - F_1 \\ m_2 a = F_2 - C \end{pmatrix} \leftarrow \text{Newton's } 2^{nd} \text{ Law for } m_1 \,\&\, m_2 \text{ (parallel to plane component)} \\ + \\ \hline &(m_1+m_2)a = F_2 - F_1 \\ &\therefore \quad a = \frac{F_2 - F_1}{m_1+m_2} \end{aligned}$$

$$C = m_1 a + F_1 = m_1\left(\frac{F_2 - F_1}{m_1+m_2}\right) + F_1 = m_1\left(\frac{F_2 - F_1}{m_1+m_2}\right) + F_1\left(\frac{m_1+m_2}{m_1+m_2}\right)$$

$$C = \frac{m_1 F_2 + F_1 m_2}{m_1+m_2} = \boxed{\frac{m_1 F_2 + m_2 F_1}{m_1+m_2}}$$

<u>Remark</u>: we should have expected the answer has 1 ⟷ 2 exchange symmetry.

**Problem 10** (10pts) Suppose a car is increasing its speed at a rate of $3m/s^2$ when its speed is $20\,m/s$. If the car is moving around a flat circular track with radius $R = 100\,m$ then what is the magnitude of the acceleration on the car ? Also, plot the acceleration vector based at the indicated dot (which represents the car) on the diagram below:

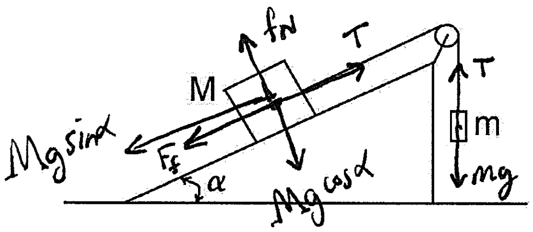


$$a_c = \frac{v^2}{R} = \frac{(20\,m/s)^2}{100\,m} = 4\,m/s^2$$

$$a = \sqrt{a_T^2 + a_c^2} = \sqrt{(3\,m/s^2)^2 + (4\,m/s^2)^2}$$

$$\boxed{a = 5\,m/s^2}$$

**Problem 11** (15pt) Suppose the plane has a coefficient of static friction of $\mu_s = 0.8$ for the box $M$. Further suppose $M = 4m$ is connected by a very light cable over an essentially massless pulley to the mass $m$. Find largest angle $\alpha$ for which the masses stay motionless. *Hint:* $\sin\alpha = \sqrt{1-\cos^2\alpha}$.



Equilbrium: $a = 0$

$$0 = mg - T \quad \therefore \quad T = mg$$

$$0 = f_N - Mg\cos\alpha$$

$$M = 4m$$

$$f_N = 4mg\cos\alpha$$

$$F_f = 0.8 f_N = 3.2 mg\cos\alpha \quad (\text{max})$$

Newton's 2nd Law $\parallel$-plane:

$$0 = T - F_f - Mg\sin\alpha$$

$$0 = mg - 3.2mg\cos\alpha - 4mg\sin\alpha$$

$$\Rightarrow \sin\alpha + 0.8\cos\alpha = 0.25$$

$$\Rightarrow \sqrt{1-\cos^2\alpha} = 0.25 - 0.8\cos\alpha$$

$$\Rightarrow \sqrt{1-x^2} = 0.25 - 0.8x$$

$$\Rightarrow 1 - x^2 = (0.25 - 0.8x)^2 = (0.25)^2 - 0.4x + 0.64x^2$$

$$\Rightarrow 1.64x^2 - 0.4x - 0.9375 = 0$$

$$x = \frac{0.4 \pm \sqrt{(0.4)^2 - 4(1.64)(-0.9375)}}{2(1.64)} \cong \underbrace{0.888}_{\text{physically interesting sol}^{\text{n}}} \text{ or } -0.644$$

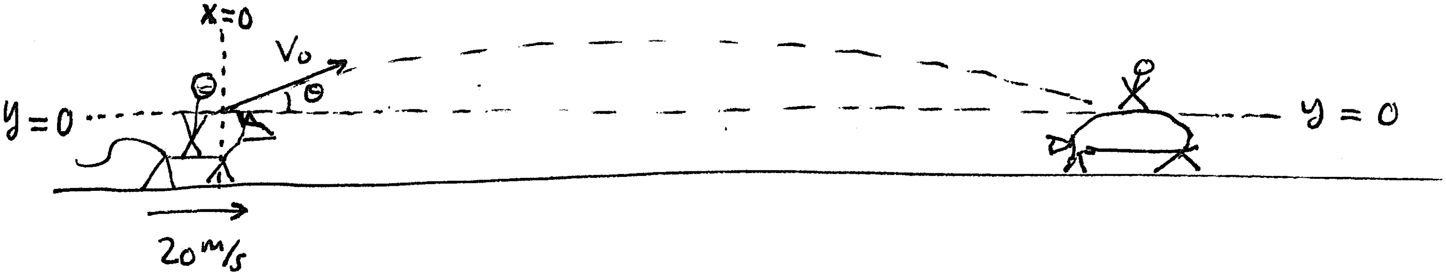
$$\cos\alpha = 0.888 \Rightarrow \alpha = \cos^{-1}(0.888) = \boxed{27.38°}$$

**Problem 12** (30pts) On level ground Kakashi can throw a particular ninja tool a distance of $50\,m$. If Kakashi throws the ninja tool while riding on a sprinting hound running $20\,m/s$ then **how far can Kakashi throw the tool?.** Ignore air resistance and assume both throws are optimal and the throws begin and end at the same height (perhaps his target is Mighty-Guy on a large turtle, or, a large cat). Also, assume Kakashi can throw just as hard when on the ground or riding the ninja hound. (use calculus as appropriate)

Max distance on level ground is given by $\theta = 45°$

then $R = \frac{V_0^2 \sin(2\theta)}{g} = \frac{V_0^2}{g} = 50m \quad \therefore \quad V_0^2 = (50m)g$

thus $V_0 = \sqrt{(50m)(9.8\,m/s^2)} = 22.14\,m/s$



$V_{0x} = V_0 \cos\theta + 20\,m/s$

$V_{0y} = V_0 \sin\theta$

tool hits target

$$y = (V_0 \sin\theta)t - \frac{1}{2}gt^2 = t\left(V_0 \sin\theta - \frac{gt}{2}\right) \overset{\swarrow}{=} 0 \Rightarrow t = \frac{2V_0 \sin\theta}{g}$$

$$x = (V_0 \cos\theta + 20\,m/s)\,t$$

Omitting units,

$$x(\theta) = (V_0 \cos\theta + 20)\left(\frac{2V_0 \sin\theta}{g}\right) \swarrow$$ now maximize it!

Thus consider the max value of $x(\theta)$ happens at critical angle,

$$\frac{dx}{d\theta} = -V_0 \sin\theta\left(\frac{2V_0 \sin\theta}{g}\right) + (V_0 \cos\theta + 20)\left(\frac{2V_0 \cos\theta}{g}\right)$$

$$= \frac{2V_0}{g}\left[-V_0 \sin^2\theta + V_0 \cos^2\theta + 20\cos\theta\right] \quad (\cos^2\theta - 1 = -\sin^2\theta)$$

$$= \frac{2V_0}{g}\left[2V_0 \cos^2\theta + 20\cos\theta - V_0\right] = 0 \quad \swarrow \lambda = \cos\theta$$

Quadratic equation $2V_0\lambda^2 + 20\lambda - V_0 = 0 \longrightarrow \lambda = 0.5165$ or $-0.9681$

We find $\theta_{max} = \cos^{-1}(0.5165) = 58.9° \Rightarrow x = (V_0 \cos(58.9°) + 20)\left(\frac{2V_0 \sin\theta}{g}\right)$

$$\boxed{x = 121.6\,m}$$