Printed Name: ______________________________.

Physics 331	Mission 1: Vectors and Calculus (37+3)pts)

Problems mentioned refer to David Griffith's *Introduction to Electrodynamics*, 5th edition. Note, I have many problems solved from the 3rd edition, but the numbering sometimes differs. Furthermore, I should warn, I have learned easier ways to solve many of those problems. For example, in this assignment the use of index notation can save much calculation. For a vector, or vector field, $\vec{F} = \langle F_1, F_2, F_3 \rangle$ we write

$$\vec{F} = F_1 \underbrace{\langle 1,0,0\rangle}_{\widehat{x_1}} + F_2 \underbrace{\langle 0,1,0\rangle}_{\widehat{x_2}} + F_3 \underbrace{\langle 0,0,1\rangle}_{\widehat{x_3}} = F_1\widehat{x_1} + F_2\widehat{x_2} + F_3\widehat{x_3} = \sum_{i=1}^{3} F_i \widehat{x_i}.$$

In this notation vector addition is defined by $(\vec{A}+\vec{B})_i = A_i + B_i$ and scalar multiplication is defined by $(c\vec{A})_i = cA_i$ for all $i = 1,2,3$. Given $\vec{A}, \vec{B}$ we have $\vec{A} = \sum_{i=1}^{3} A_i \widehat{x_i}$ and $\vec{B} = \sum_{i=1}^{3} B_i \widehat{x_i}$. Moreover,

$$\vec{A} \bullet \vec{B} = \sum_{i=1}^{3} A_i B_i \qquad \& \qquad \vec{A} \times \vec{B} = \sum_{i,j,k=1}^{3} \epsilon_{ijk} A_i B_j \widehat{x_k}.$$

Gradient of f, divergence of $\vec{F}$ and curl of $\vec{F}$ are likewise defined by

$$\nabla f = \sum_{i=1}^{3} \frac{\partial f}{\partial x_i}\widehat{x_i} \qquad \& \qquad \nabla \bullet \vec{F} = \sum_{i=1}^{3} \frac{\partial F_i}{\partial x_i} \qquad \& \qquad \nabla \times \vec{F} = \sum_{i,j,k=1}^{3} \epsilon_{ijk} \frac{\partial F_j}{\partial x_i}\widehat{x_k}.$$

Or, using the notation $\frac{\partial}{\partial x_i} = \partial_i$ we have $\nabla f = \sum_{i=1}^{3} (\partial_i f)\widehat{x_i}$ and $\nabla \bullet \vec{F} = \sum_{i=1}^{3} \partial_i F_i$ and $\nabla \times \vec{F} = \sum_{i,j,k=1}^{3} \epsilon_{ijk} (\partial_i F_j)\widehat{x_k}$. I should mention, when faced with ϵ_{ijk} symbols contracted against one another there is a well-known[1] identity which is exceedingly helpful: $\sum_k \epsilon_{mnk}\epsilon_{ijk} = \delta_{mi}\delta_{nj} - \delta_{mj}\delta_{ni}$.

Advice: these problems are not to be done in one day. I strongly recommend reading all of them, or at least keep reading until you see one that you feel confident you can solve. Work that problem. Work problems on separate pages so you have no issue of turning them in order. Please only work on one side for the sake of my grading. Then, go on, find another problem you can devote to destruction. Continue, until you find the core problems which mock and malign your attacks. Sleep on those. Try them again. Rinse, repeat. Of course, you are welcome to ask for hints at any time. I will try to build hints into lecture as well. This particular Mission is more broad than most so the difficulty is probably not indicative of later problem sets. It is both harder and easier than those. Finally, keep in mind the solutions I have posted, there are solutions to some of these problems. I also don't mind you looking up solutions to the Griffith's problems, it's not difficult to find instructors solution manuals. The point of these is for you to relearn Calculus III, how to use vector calculus and the very new parts of curvelinear vector calculus and the Dirac Delta function.

Problem 1 (3pts) This problem is based in part on Problems 1.5 & 1.6.

(a.) Show $\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \left(\vec{A} \bullet \vec{C}\right)\vec{B} - \left(\vec{A} \bullet \vec{B}\right)\vec{C}$. Note: this is often known as the *BAC-CAB* identity since that is how it is written in the front-cover of the text.

[1] within the context of people who face such problems

(b.) Show $\vec{A} \times \left(\vec{B} \times \vec{C}\right) + \vec{B} \times \left(\vec{C} \times \vec{A}\right) + \vec{C} \times \left(\vec{A} \times \vec{B}\right) = 0$. Note: this is the Jacobi identity, the fact that the cross-product satisfies this rule paired with it's linearity shows $\mathbb{R}^3$ paired with $\times$ forms a *Lie Algebra*. Lie algebras are of central importance to modern particle physics.

(c.) When is it true that $\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \left(\vec{A} \times \vec{B}\right) \times \vec{C}$?

Problem 2 (4pts) This problem based in part on Griffith's Problem 1.21 & 1.24. I give a bit more guidance.

(a.) Let $\vec{A}, \vec{B}$ and f be smooth. Show,

(i.) $\nabla \bullet (f\vec{A}) = (\nabla f) \bullet \vec{A} + f(\nabla \bullet \vec{A})$

(ii.) $\nabla \times (f\vec{A}) = (\nabla f) \times \vec{A} + f(\nabla \times \vec{A})$

(b.) Let g be smooth and nonzero and let $\vec{B} = \frac{\vec{A}}{g}$ hence $\vec{A} = g\vec{B}$.

(i.) Derive the quotient rule for divergence from $\nabla \bullet \vec{A} = \nabla \bullet (g\vec{B})$.

(ii.) Derive the quotient rule for curl from $\nabla \times \vec{A} = \nabla \times (g\vec{B})$.

Problem 3 (3pts) Problem 1.13 (calculus of the displacement vector). We define $\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$ where $\vec{r} = (x, y, z)$ and $\vec{r}\,' = (x', y', z')$. Here we consider $\vec{r}$ as a variable point and $\vec{r}\,'$ as fixed.

(a.) Show that: $\nabla(\mathscr{r}^2) = 2\vec{\mathscr{r}}$

(b.) Show that: $\nabla\left(\frac{1}{\mathscr{r}}\right) = \frac{-\hat{\mathscr{r}}}{\mathscr{r}^2}$

(c.) Find the formula for $\nabla(\mathscr{r}^n)$

Problem 4 (2pts) Problem 1.36. I diverge notationally from Griffith's in that I prefer to use ∂S to denote the positively oriented boundary of the surface S and $\partial \mathcal{V}$ to denote the positively oriented boundary of the volume $\mathcal{V}$.

(a.) Show that $\displaystyle\int_S f(\nabla \times \vec{A}) \bullet d\vec{a} = \int_S \left[\vec{A} \times (\nabla f)\right] \bullet d\vec{a} + \oint_{\partial S} f\vec{A} \bullet d\vec{l}$

(b.) Show that $\displaystyle\int_{\mathcal{V}} \vec{B} \bullet (\nabla \times \vec{A}) d\tau = \int_{\mathcal{V}} \vec{A} \bullet (\nabla \times \vec{B}) d\tau + \oint_{\partial \mathcal{V}} (\vec{A} \times \vec{B}) \bullet d\vec{a}$.

Problem 5 (3pt) Problem 1.38

Problem 6 (3pt) Problem 1.39

Problem 7 (3pt) Problem 1.44a & 1.45a & 1.46

Problem 8 (3pt) Problem 1.47

Problem 9 (3pt) Problem 1.50

Problem 10 (3pt) Problem 1.54

Problem 11 (3pt) Problem 1.59

Problem 12 (4pt) Problem 1.61

Problem 13 (3pt) Problem 1.63