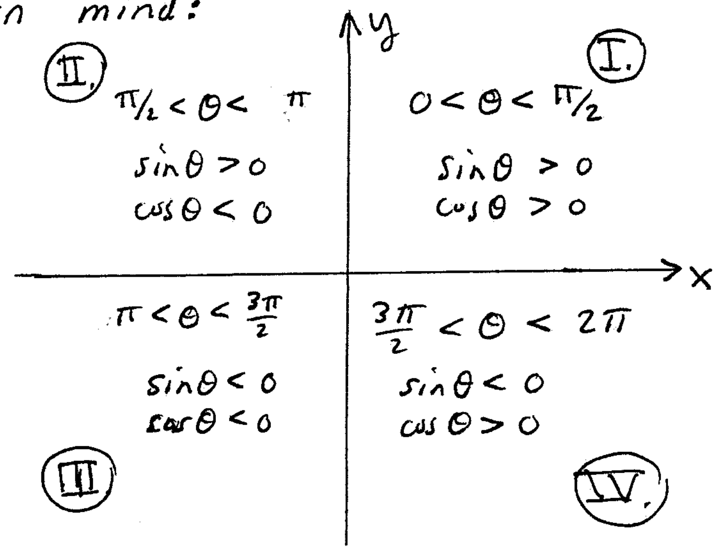
Polar Coordinates:

Polar coordinates r, θ are related to Cartesian coordinates x, y by the equations,

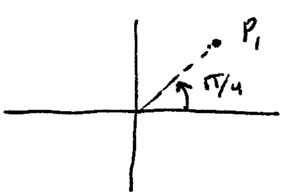
$$\boxed{\begin{array}{lcl} x = r\cos\theta & & x^2 + y^2 = r^2 \\ y = r\sin\theta & & \tan\theta = y/x \quad \text{for } x \neq 0 \end{array}}$$

Here θ is measured counter-clock-wise (CCW) from the positive x-axis, naturally θ and $\theta + 2\pi j$ for $j \in \mathbb{Z}$ describe same ray from the origin. Also, to be picky the origin $(0,0)$ has $r = 0$ and θ undefined. Some texts allow $r < 0$ others insist $r \geq 0$. You'll have to judge by context. Finding <u>standard angle</u> θ is a bit tricky. We must keep in mind:



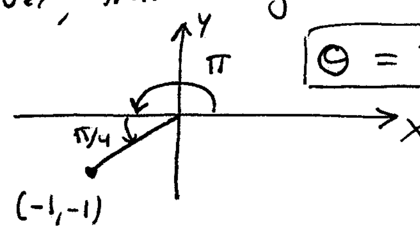
Remark:
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$
only legit for $x > 0$

$\boxed{E1}$ $P_1 = (2,2)$
has $\theta = \tan^{-1}\left(\frac{2}{2}\right) = \frac{\pi}{4}$



$\boxed{E2}$ $P_2 = (-1,-1)$
has $\tan^{-1}\left(\frac{-1}{-1}\right) = \tan^{-1}(1) = \frac{\pi}{4}$
However, from diagram we see

$$\boxed{\theta = \pi + \frac{\pi}{4}}$$



Converting back and forth from CARTESIANS TO POLARS

E3 $x^2 + y^2 = 9 \Rightarrow r^2 = 9 \Rightarrow \boxed{r = 3}$

polar form of circle centered at origin

E4 $y = mx + b$

$r\sin\theta = mr\cos\theta + b$

$r(\sin\theta - m\cos\theta) = b$

$$\boxed{r = \frac{b}{\sin\theta - m\cos\theta}}$$

(line recast as a polar graph; $r = f(\theta)$)

E5 $y = A(x - x_0)^2 + y_0$ is parabola with vertex (x_0, y_0)

In polar coordinate,

$$\boxed{r\sin\theta = A(r\cos\theta - x_0)^2 + y_0}$$

E6 $y = x^2 + 1$

$r\sin\theta = (r\cos\theta)^2 + 1 = r^2\cos^2\theta + 1$

$$\boxed{r\sin\theta - r^2\cos^2\theta = 1}$$

E7 $y = x^2 \longrightarrow r\sin\theta = r^2\cos^2\theta$

$$\Rightarrow \boxed{r = \frac{\sin\theta}{\cos^2\theta}} = \frac{\sin\theta}{1 - \sin^2\theta}$$

E8 The sol^{n} set of $r = \dfrac{ed}{1+e\cos\theta}$ forms a <u>conic section</u> with <u>eccentricity</u> e. (see * for another formulation)

$e<1$: ellipse

$e=1$: parabola

$e>1$: hyperbola

Let's attempt changing to Cartesian coord, e, d constants,

$$r + er\cos\theta = ed$$

$$\sqrt{x^2+y^2} = ed - er\cos\theta = e(d-x)$$

$$x^2+y^2 = e^2(d-x)^2 = e^2(d^2 - 2dx + x^2)$$

$$(1-e^2)x^2 + 2e^2dx + y^2 = e^2d^2$$

$$\left(\frac{1}{e^2}-1\right)x^2 + 2dx + y^2/e^2 = d^2$$

assuming $e \neq 1$

$$x^2 + \left[\frac{2d}{\frac{1}{e^2}-1}\right]x + \frac{1}{e^2\left(\frac{1}{e^2}-1\right)}y^2 = \frac{d^2}{\frac{1}{e^2}-1}$$

$$x^2 + \frac{2e^2d}{1-e^2}x + \frac{1}{1-e^2}y^2 = \frac{e^2d^2}{1-e^2}$$

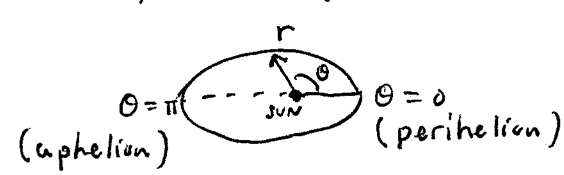
$$\underbrace{\left(x - \frac{e^2d}{1-e^2}\right)^2 + \left(\frac{1}{1-e^2}\right)y^2 = \frac{e^2d^2}{1-e^2} - \left[\frac{e^2d}{1-e^2}\right]^2}$$

<u>Better yet</u>,

• You can see how $e<1$ vs. $e>1$ changes the game.

Polar Eq^{n} of ellipse with focus at origin, semi-major axis a and eccentricity e,

(*) $\boxed{r = \dfrac{a(1-e^2)}{1+e\cos\theta}}$



Here we see better $a = R,\ e = 0$ gives $r = R$.

Polar Curve Graphing

1.) given $r = f(\theta)$ plot graph in .

2.) translate to xy-plane.

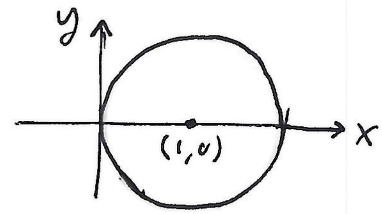
3.) if possible avoid 1 & 2 by algebra!

$\boxed{E9}$ $r = 2\cos\theta$

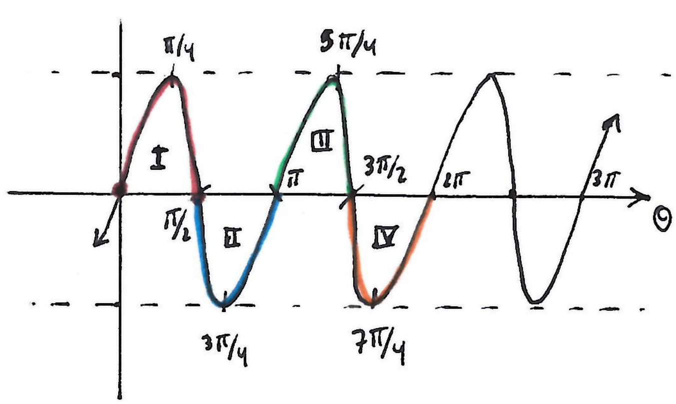
$r^2 = 2r\cos\theta \quad \therefore \quad x^2 + y^2 = 2x$

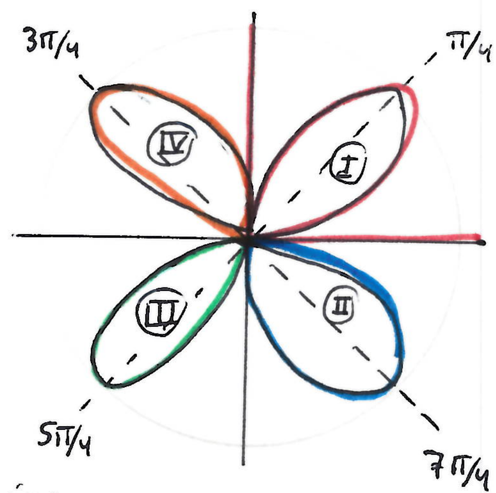
$$\underbrace{(x-1)^2 + y^2 = 1}$$

CIRCLE, radius 1, centered at $(1,0)$



$\boxed{E10}$ $r = 3\sin(2\theta)$





• (we allow $r < 0$ in such graphing with understanding $(r, \theta) \longleftrightarrow (-r, \theta + \pi)$).