MA 141: Section 3.5 (Chain Rule)

A. Find $f'(x)$.

STRATEGY: Figure out how to write $f(x) = g(u(x))$ so we can apply chain rule

1. $f(x) = \sin(x^2 + 2x - 3)$ make $u = x^2 + 2x - 3$ and $g(u) = \sin(u)$

$$\frac{df}{dx} = \frac{d}{dx}\left(g(u(x))\right) = \frac{dg}{du}\frac{du}{dx} = \cos(u)\,(2x+2) = \boxed{(2x+2)\cos(x^2+2x-3)}$$

2. $f(x) = \sqrt[4]{-2x^2 + 4x + 7}$ make $u = -2x^2 + 4x + 7$ and $g(u) = u^{1/4}$

$$\frac{d}{dx}\left(\sqrt[4]{-2x^2+4x+7}\right) = \frac{dg}{du}\frac{du}{dx} = \frac{1}{4}u^{-3/4}(-4x+4) = \boxed{(1-x)(-2x^2+4x+7)^{-3/4}}$$

3. $f(x) = \sqrt[5]{5x^2 - 17x}$ make $u = 5x^2 - 17x$ and $g(u) = u^{1/5}$

$$\frac{d}{dx}\left(\sqrt[5]{5x^2-17x}\right) = \frac{dg}{du}\frac{du}{dx} = \frac{1}{5}u^{-4/5}(10x-17) = \boxed{\left(\frac{10x-17}{5}\right)(5x^2-17x)^{-4/5}}$$

4. $f(x) = \sin(\sin(x))$ make $u = \sin x$ and $g(u) = \sin(u)$

$$\frac{d}{dx}\left(\sin(\sin(x))\right) = \frac{dg}{du}\frac{du}{dx} = \cos(u)\cos(x) = \boxed{\cos(\sin(x))\cos(x)}$$

5. $f(x) = \dfrac{1}{\cos^3(x)}$ make $u = \cos(x)$ and $g(u) = \frac{1}{u^3} = u^{-3}$

$$\frac{d}{dx}\left((\cos(x))^{-3}\right) = \frac{dg}{du}\frac{du}{dx} = -3u^{-4}(-\sin(x)) = \boxed{\frac{3\sin(x)}{\cos^4(x)}}$$

6. $f(x) = \sin^2(x) + \cos^2(x) = 1$

$$\frac{d}{dx}\left(f(x)\right) = \frac{d}{dx}(1) = \boxed{0}$$

$$\frac{d}{dx}\left([\cos(2x)]^2\right) = \frac{dh}{du_1}\frac{du_1}{dx} \leftarrow \quad h(u_1) = u_1^2,\ u_1 = \cos(2x)$$

$$= 2u_1\frac{d}{dx}(\cos 2x) \quad P(u_2) = \cos u_2,\ u_2 = 2x$$

$$= 2u_1\frac{dP}{du_2}\frac{du_2}{dx}$$

$$= 2u_1(-\sin u_2)2$$

$$= -4\cos(2x)\sin(2x)$$

Use more paper than me and it's easier ☺ →

7. $f(x) = e^{\tan(x)} + \cos^2(2x)$ let $u = \tan(x)$, $g(u) = e^u$

$$\frac{d}{dx}\left(e^{\tan(x)}\right) = \frac{dg}{du}\frac{du}{dx} = e^u\sec^2(x) = e^{\tan(x)}\sec^2(x)$$

Now piece these together $\frac{df}{dx} = \boxed{e^{\tan(x)}\sec^2(x) - 4\cos(2x)\sin(2x)}$

8. $f(x) = \sqrt{\sin(x)}$ let $u_1 = \sin(x)$ and $g(u_1) = \sqrt{u_1}$

$$\frac{d}{dx}\left(\sqrt{\sin(x)}\right) = \frac{dg}{du_1}\frac{du_1}{dx} = \frac{1}{2\sqrt{u_1}}\cos(x) = \boxed{\frac{\cos(x)}{2\sqrt{\sin(x)}}}$$

Note: I use the chain rule in most of these problems, there are some steps missing if you don't understand some step please make a note and ask monday in the review.

B. Find the derivatives of the following functions.

1. $\frac{d}{dt}\left(\frac{5}{\sqrt{t^3+t^2+7t}}\right) = 5\left(\frac{-1}{2}(t^3+t^2+7t)^{-3/2}\right)\frac{d}{dt}(t^3+t^2+7t) = \boxed{\frac{-5}{2}(t^3+t^2+7t)^{-3/2}(3t^2+2t+7)}$

2. $\frac{d}{dx}\left(e^{5x}\ln(x^2+9)\right) = \frac{d}{dx}(e^{5x})\ln(x^2+9) + e^{5x}\frac{d}{dx}(\ln(x^2+9))$
$= 5e^{5x}\ln(x^2+9) + e^{5x}\frac{1}{x^2+9}\frac{d}{dx}(x^2+9) = \boxed{e^{5x}\left(5\ln(x^2+9)+\frac{2x}{x^2+9}\right)}$

3. $\frac{d}{dx}\left(\frac{\cos(3x+7)}{x^5+x^3+9}\right) = \frac{\frac{d}{dx}(\cos(3x+7))(x^5+x^3+9) - \cos(3x+7)\frac{d}{dx}(x^5+x^3+9)}{(x^5+x^3+9)^2}$
$= \boxed{\frac{-3\sin(3x+7)(x^5+x^3+9) - \cos(3x+7)(5x^4+3x^2)}{(x^5+x^3+9)^2}}$

4. $\frac{d}{dt}\left(\sec(\ln(t))\right) = \sec(\ln(t))\tan(\ln(t))\frac{d}{dt}(\ln(t)) = \boxed{\frac{\sec(\ln(t))\tan(\ln(t))}{t}}$

5. $\frac{d}{dy}\left(\sin^5(2y+7)\right) = 5\sin^4(2y+7)\cdot\frac{d}{dy}(2y+7) = \boxed{10\sin^4(2y+7)}$

6. $\frac{d}{dx}\left(\frac{2x+1}{\sec(5x)}\right) = \frac{2(\sec(5x)) - (2x+1)\frac{d}{dx}(\sec(5x))}{\sec^2(5x)}$
$= \frac{2\sec(5x) - (2x+1)\sec(5x)\tan(5x)\cdot 5}{\sec^2(5x)} = \boxed{\frac{2-5(2x+1)\tan(5x)}{\sec(5x)}}$

7. $\frac{d}{dx}\left((x-5)^3(x+5)^9\right) = \left[\frac{d}{dx}(x-5)^3\right](x+5)^9 + (x-5)^3\left[\frac{d}{dx}(x+5)^9\right]$
$= \boxed{3(x-5)^2(x+5)^9 + 9(x-5)^3(x+5)^8}$

8. $\frac{d}{dx}\left(\frac{5}{x^2+1}+\frac{4}{x}\right) = 5\frac{d}{dx}\left[(x^2+1)^{-1}\right] + 4\frac{d}{dx}(x^{-1})$
$= -5(x^2+1)^{-2}\cdot 2x - 4x^{-2} = \boxed{\frac{-10x}{(x^2+1)^2} - \frac{4}{x^2}}$

9. $\frac{d}{dx}\left(\frac{e^{2x}}{1-e^{5x}}\right) = \frac{\frac{d}{dx}(e^{2x})(1-e^{5x}) - e^{2x}\frac{d}{dx}(1-e^{5x})}{(1-e^{5x})^2} = \frac{2e^{2x}(1-e^{5x}) - e^{2x}(-5e^{5x})}{(1-e^{5x})^2}$
$= \boxed{\frac{e^{2x}(2+3e^{5x})}{(1-e^{5x})^2}}$

10. $\frac{d}{du}\left(\ln(e^{u^2})\right) = \frac{d}{du}(u^2) = \boxed{2u}$